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MEASUREMENT OF SHIP MODEL WAKE VARIATION IN ROUGH WATER

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MEASUREMENT OF SHIP MODEL
WAKE VARIATION IN ROUGH WATER

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MEASUREMENT OF SHIP MODEL WAKE VARIATION IN ROUGH WATER, by JOHN D. O'CONNEL and EVENSON M. BURTIS. Submitted to the Department of Naval Architecture and Marine Engineering on 21 May 1960 in partial fulfillment of the requirements for the degree of Naval Engineer and the degree of Master of Science in Naval Architecture and Marine Engineering.

ABSTRACT

The object of this investigation was to devise instrumentation for a small ship model to accurately measure the variables considered significant in the variation of wake velocity of a ship operating in rough water. It was hoped that by analysis of the results of tests at the M.I.T. Towing Tank to show conclusively that wake variation for a model towed in ahead seas was caused by the orbital velocity of the water particles in the waves.

To accurately fix the cause of wake variation, it was necessary to measure the motion of the model in the vicinity of the propeller location, and to locate the position of the wave crest at the times when the wake velocity was at its maximum and minimum points.

Results from the tests run were inconclusive as to the exact cause of the wake variation. The theory that orbital velocity of the water particles affects the magnitude of the wake has been supported. In addition, evidence is presented that the motion of the stern, particularly that caused by pitching, is also a major contributor to the variation of wake.

Additional work is required to substantiate these results, with particular attention necessary to avoid unnecessary phase lags in the measurement systems. It would be desirable to devise a means for measuring the instantaneous fluid velocity in the vicinity of the propeller that did not involve the relatively large inertia and resistance of the entrained water in any rotating propeller type mechanism.

Thesis Supervisor: S. Curtis Powell

Title: Associate Professor of Marine Engineering

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TABLE OF CONTENTS

TABLE OF CONTENTS (con'd)

LIST OF FIGURES

LIST OF FIGURES

APPENDIX

LIST OF TABLES

APPENDIX

I.

INTRODUCTION

The following quotation from Powell (12)* best sums up the need for the experimental work conducted as part of this thesis:

> "In recent years considerable experimental work relating to ship resistance and motion under adverse weather conditions has been carried out at many model facilities. Projections for model behavior to that of the full size ship have been made by numerous investigators, guided by the considerable contemporary advances in dynamic, hydrodynamic, and oceanographic theory and knowledge. Such projections or analyses of full scale observations require that the effect of adverse weather on those parameters which determine propeller operating conditions and propulsive coefficient be taken into account. Experiment and observation bearing on these are not nearly as extensive as for resistance and ship motion effects."

Propeller design and prediction of propeller performance is largely based on the concept of wake fraction, which relates the speed of the ship to the actual relative water velocity to which the propeller is subjected. Elementary fluid dynamics shows that a fluid moving past any object

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* Numbers in () refer to List of References in Appendix H.

is affected both in magnitude and direction by the presence of that object. In the steady state, i.e. calm water and steady ship's speed, this accounts for the wake fraction.

When a ship is operating in rough water, conditions can no longer be considered steady. In the first place the ship, which had been assumed to be moving with a steady velocity in one direction only, is now subjected to motion in six degrees of freedom caused by the action of the waves. If consideration is limited to the case of regular seas from directly ahead, the propeller still has some oscillatory motion in the vertical plane superimposed on its speed of advance in the horizontal plane.

In addition, the trocoidal theory of wave motion shows that at any given instant of time there is an orbital velocity of the water particles, dependent in magnitude on wave length, wave height and depth below the surface. The direction of this water velocity is dependent on the location of the wave crests and troughs.

Thus it may be theorized that, where in calm water the relative velocity between propeller and water is dependent only on the speed of the ship and the hull form, both of which are constant for any given operating condition, the relative velocity between propeller and water when a ship is operating in waves is a function of the calm water variables plus pitching and heaving motion and the location

and size of the waves with respect to the location of the propeller.

The most extensive work to date on this more complicated picture has been performed by Norley (11) at the David Taylor Model Basin. This series of experiments was conducted in regular waves with the propeller model in open water, and was an investigation into the variation of thrust and torque of a propeller exposed to waves. Norley's results strongly indicated that the axial velocities encountered by the propeller varied with the orbital velocities of the waves. The principal limitation to these results is that this was an open water test, with the propeller shaft held motionless. Any effect of ship's hull or motion in the vertical plane was not taken into account.

Two rough preliminary efforts to include these latter two effects were conducted at M.I.T. by Hoeflein (7) and Metzger (10). Their results were so sketchy as to serve only as slight confirmation that wake velocity did fluctuate in rough water. These efforts were further hindered by the fact that they attempted to correlate wake variation with model speed. It has been found that model speed varies so little from its mean value under rough water conditions that the relatively large wake variations could in no way be accounted for by this means alone. An attempt was made to compare the magnitude of the wake variation with

the orbital velocities encountered, but this was done without including possible effects of wave position or model motion.

It was on this groundwork that the objectives of this thesis were established. It was hoped to provide answers for the following questions:

(1) Does the magnitude of the wake velocity vary when the propeller, located in its normal position, is operated in rough water, and can this variation be measured using a relatively small (5 feet) model?

(2) Is this variation, if it is present, caused by the orbital velocity of the water particles in the waves?

(3) Does the motion of the model, either pitching or heaving, have any effect on the wake velocity?

(4) Is it possible, using experimental results from small model tests, to accurately predict the variation in wake velocity of a full scale propeller mounted on a ship?

To answer these questions, more extensive apparatus than has previously been used was necessary. To completely cover all possibilities, the following variables must be measured:

(1) Model speed.

(2) Wake velocity (i.e. relative velocity of the water in a direction perpendicular to the plane of the propeller).

(3) Model motion.

(4) Position of the wave crest in relation to the position of the model (and propeller).

The requirements just proposed set the limits on the investigation that follows. Another requirement is that the procedure be adaptable to facilities and equipment available in the M.I.T. Towing Tank. This introduces the first limitation that the model size be no more than five feet long. Thus the size of the instrument package to be carried in the model is severely limited.

The motion of a model, even in regular ahead seas, is an extremely complex phenomena. To be completely exact, a system similar to that proposed by Haughey and DeLaat (6) would be required. If the analysis is limited to pitch and heave only, a simpler system is possible. The degree of simplification was set by the recording apparatus available.

The requirement that the position of the wave crest in relation to the model be accurately fixed requires that the wave recording device be in some manner connected to the motion of the model. Existing wave height recording devices at the M.I.T. Model Tank are fixed to the side of the tank,

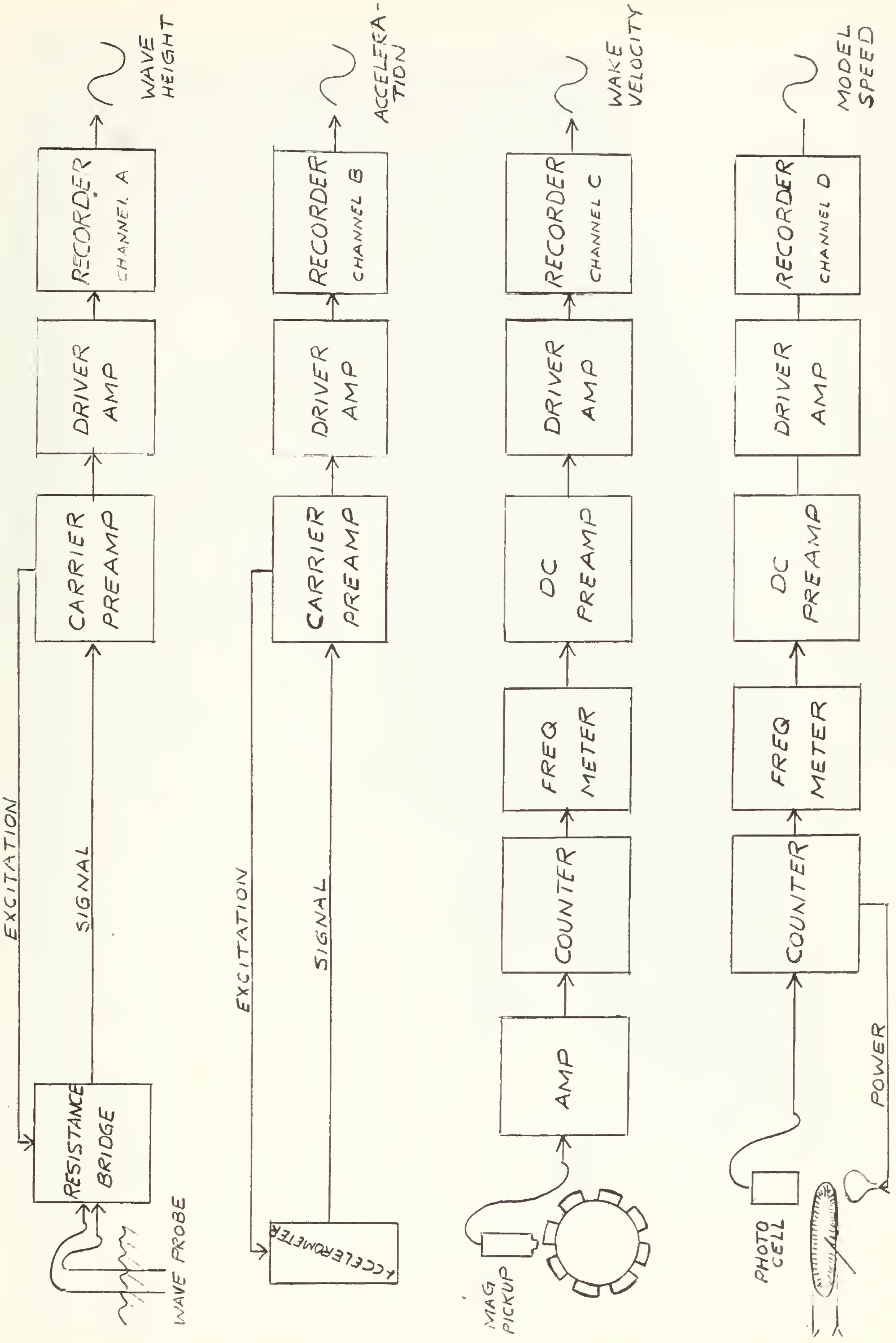
and are too large and bulky to be mounted on the model.

It therefore was the intention of the authors to devise instrumentation for a small model to accurately measure the variables considered significant in wake variation, and by analysis of the results to show conclusively that previous work showing that wake variation is a result of the orbital velocities of the waves encountered is valid.

and are too large and bulky to be mounted on the model.

It therefore was the intention of this study to devise instrumentation for a small model to measure all necessary variables considered significant to wake variation, and by analysis of the results to show conclusively that previous work showing that wake variation is a result of the orbital velocities of the waves encountered is valid.

FIGURE I



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3/60

Figure II

Ship Model with Instrumentation Installed

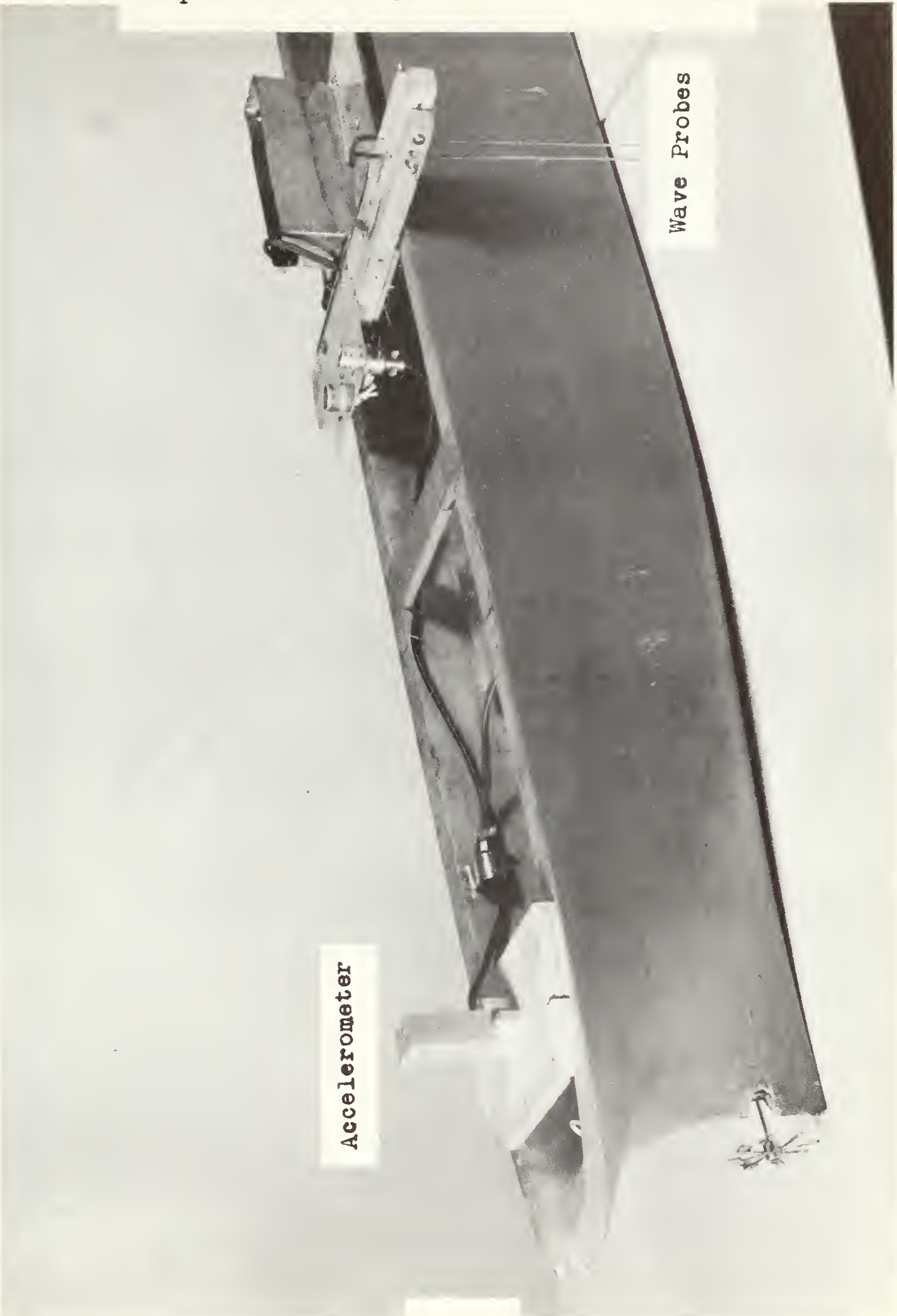


Figure III

Ship Model with Instrumentation Installed

PROCEDURE

A. General Considerations

As discussed in the Introduction, the system to be developed must be capable of measuring four separate quantities and recording them on a common time scale. Because the Ship Model Towing Tank facility at M.I.T. is presently equipped with a four-channel Sanborn Recorder it was decided to use this as the recording device, and to devise the other instrumentation with outputs compatable with the Sanborn system.

A block diagram of the final instrumentation scheme is shown in Figure I, and photographs of the ship model with equipment installed are shown in Figures II and III.

B. Wake Velocity Measurements

Since the purpose of this project was to investigate the variations in wake velocity, the first instrumentation problem undertaken was to develop a system to measure the velocity of the fluid flowing past the stern of a ship model. To make the device sensitive to minute changes in an average velocity that was in itself quite small (on the order of 2

to 3.5 feet per second) it was imperative to select an instrument that possessed very low physical inertia and a small frictional resistance.

The use of thermistors, which, like a hot wire anemometer, will give a voltage output proportional to the flow of fluid past them was considered, but this concept was discarded because of the non-directional properties of this type of measurement. Also, it was decided, after consultation with the thesis supervisor, that the integrated or average wake velocity variation was the quantity of interest rather than the velocity at a single point. The interest in this investigation lies in the effect the wake variations have on propeller performance, and it is obvious that the propeller "sees" the average velocity rather than the velocity at any particular point. It was also taken into consideration that water velocity measurements using thermistors is a new concept and much preliminary developmental work and calibration would be necessary before these devices could be used with the required degree of accuracy.

Since the main point of interest in the investigation was the effect of wake variation on propeller performance, it was decided to use a propeller-type instrument to measure the wake velocity. This type of system has been used in previous investigations of this type (7,10), and it is known to respond to the variations encountered when using a 5 ft.

ship model. However, efforts were made to improve the efficiency of the device.

It is known from experience that a freely mounted propeller will turn when immersed in a moving stream of fluid, and the angular velocity of the propeller will be roughly proportional to the velocity of the stream. It remains, however, to be able to measure this resulting angular velocity in such a manner as to not affect the response of the propeller itself. There are several ways to measure angular velocity, including:

1. Mechanical tachometer.
2. Photoelectric tachometer.
3. Metallic proximity detection device coupled with a counter and integrator.
4. Magnetic pickup coupled with a counter and integrator.

Of the above listed devices and systems, the magnetic pickup was finally selected for use for the following reasons:

1. The small driving forces that would be encountered (wake velocity acting on the propeller or vane wheel) would be unable to overcome the large physical load imposed on the shaft by a mechanical tachometer.

2. A photoelectric tachometer using a photocell and a slotted disc similar to that described in (1) would impose negligible inertial load on the shaft, but it does

require more space and an additional power lead to the model. The space available in the stern section of the model severely limited the size of disc that could be used and also presented a major problem in trying to locate a light source and photocell so that the counting circuit could be utilized. These facts brought about the decision to not use this type of device even though it does have some very good points. A metallic gear of the type used for the magnetic pickup does have more inertia than a slotted disc, but it is believed that this added inertia is not significant.

3. A proximity transducer system such as the "Electro" Model 4912-AN also was considered but was eliminated for monetary reasons. For further such investigations this type of system should be given serious consideration. The major advantage of a proximity transducer is that no long shaft and disc and gear are needed. Instead, the proximity pickup could be mounted near the vane wheel (some models are completely waterproof) and propeller RPM could be measured directly. This method would also eliminate the problem of leakage around the shaft with resultant need for friction causing shaft seals. A further refinement of this system is available in the "Electro-Tach" Model 7102 which measures shaft revolutions without physical loading (with a proximity transducer) and offers direct reading with an accuracy of $\pm$ 1%.

4. After the above systems had been considered and rejected for one or more of the listed reasons, a magnetic pickup counting system was selected for use. The unit used is shown in Figures IV and V and operates as follows:

a. A magnetic gear wheel is mounted on the inboard end of the vane wheel shaft, and the magnetic pickup is so located that its pole piece is directly above the gear teeth.

b. As the gear teeth move past the pole piece a small voltage pulse is induced in the pickup coil.

c. The pulse so generated is then amplified (if necessary) and sent to the counting units.

d. The output of the frequency meter, a D.C. voltage signal proportional to RPM, is the input to the Sanborn Recorder.

At first, an "Electro" Model 3055 Subminiature Magnetic Pickup (18) was selected because of its small size (1 inch long x 9/32 inch diameter), but as work progressed it was found that the signal to noise ratio of this unit was too low. To eliminate this problem, the Model 3055 was replaced by an "Electro" Model 3030 Magnetic Pickup (18). This unit was quite a bit larger ($2\frac{1}{4}$ inches long x 3/4 inch diameter), but it did provide a much stronger signal (Figure G-I).

In order to increase the signal strength and make up for attenuation in the approximately 100 feet of cable from

Figure IV

Propeller Assembly

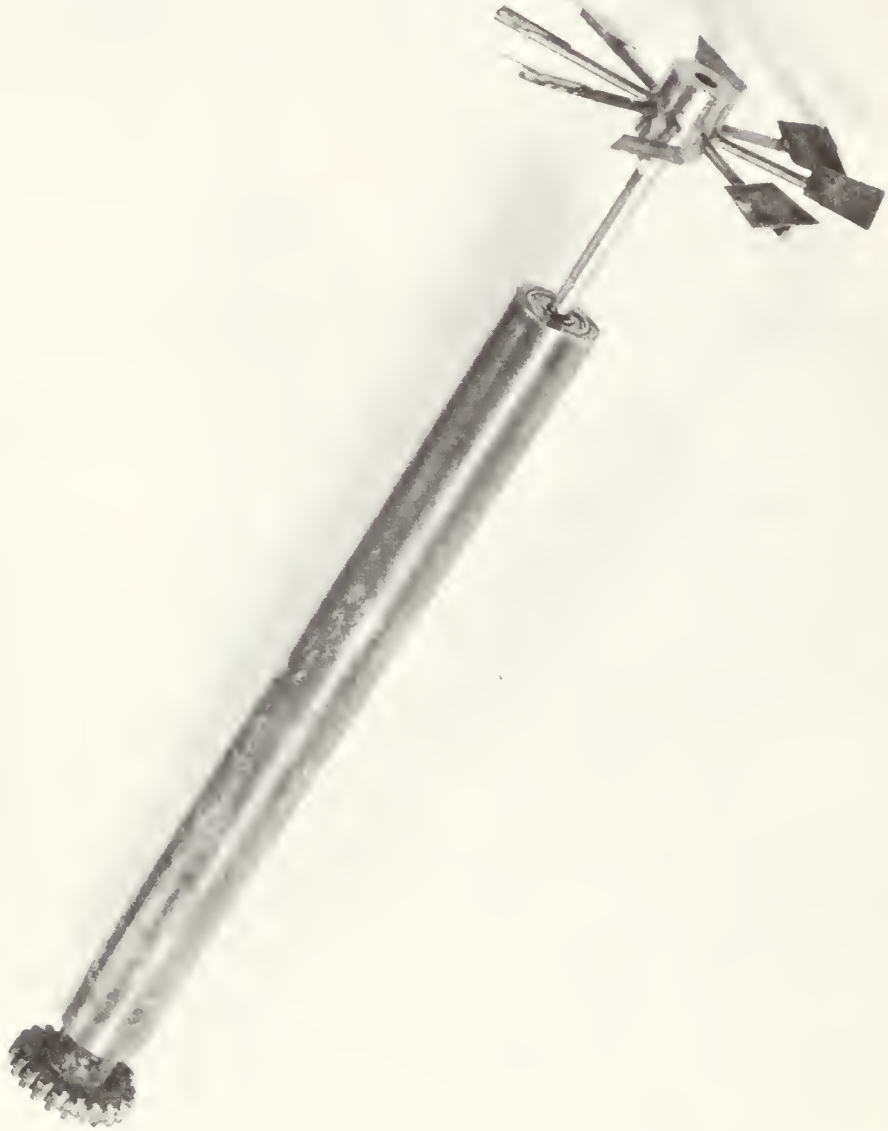


FIGURE V

Propeller Assembly
(Scale Drawing)

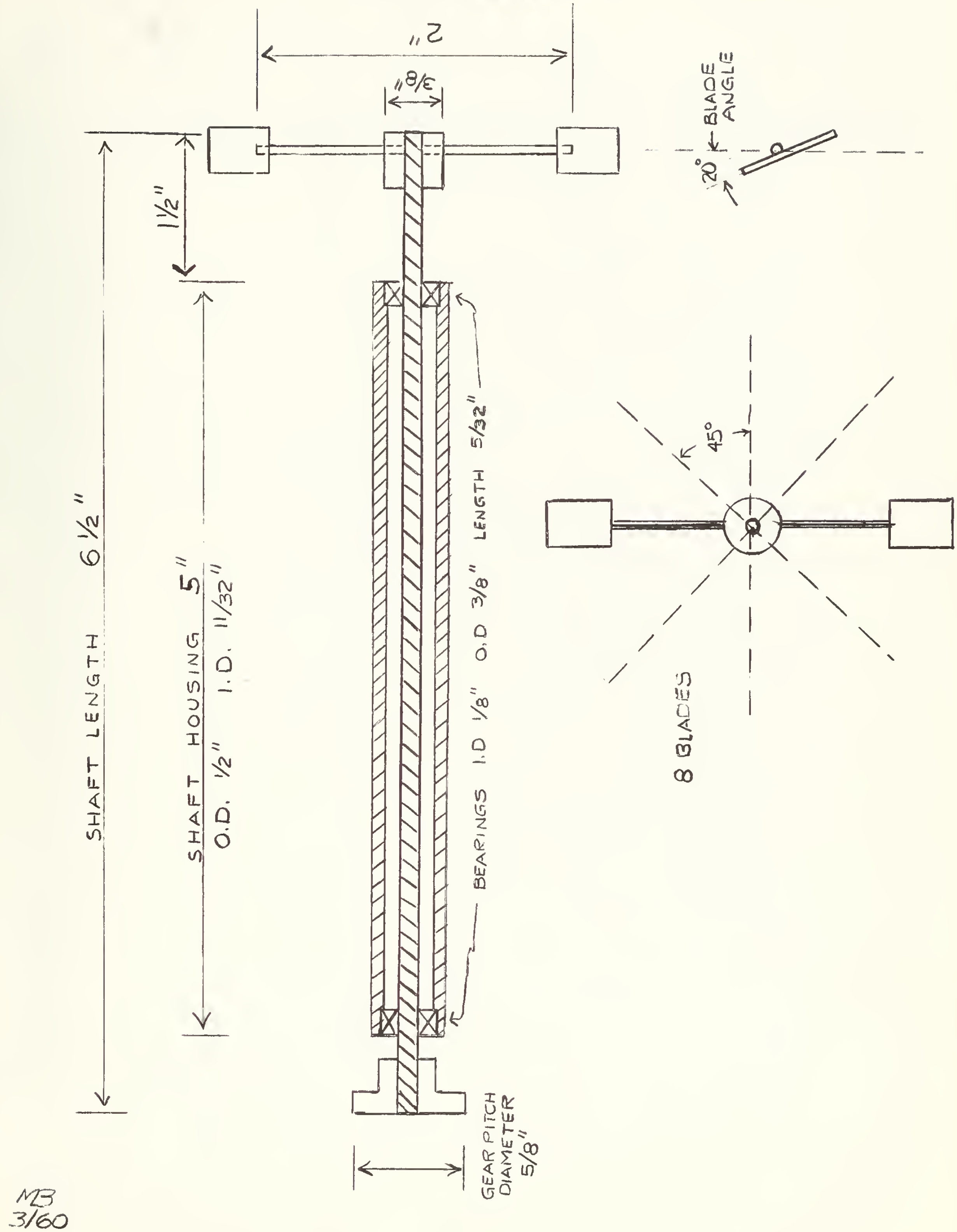
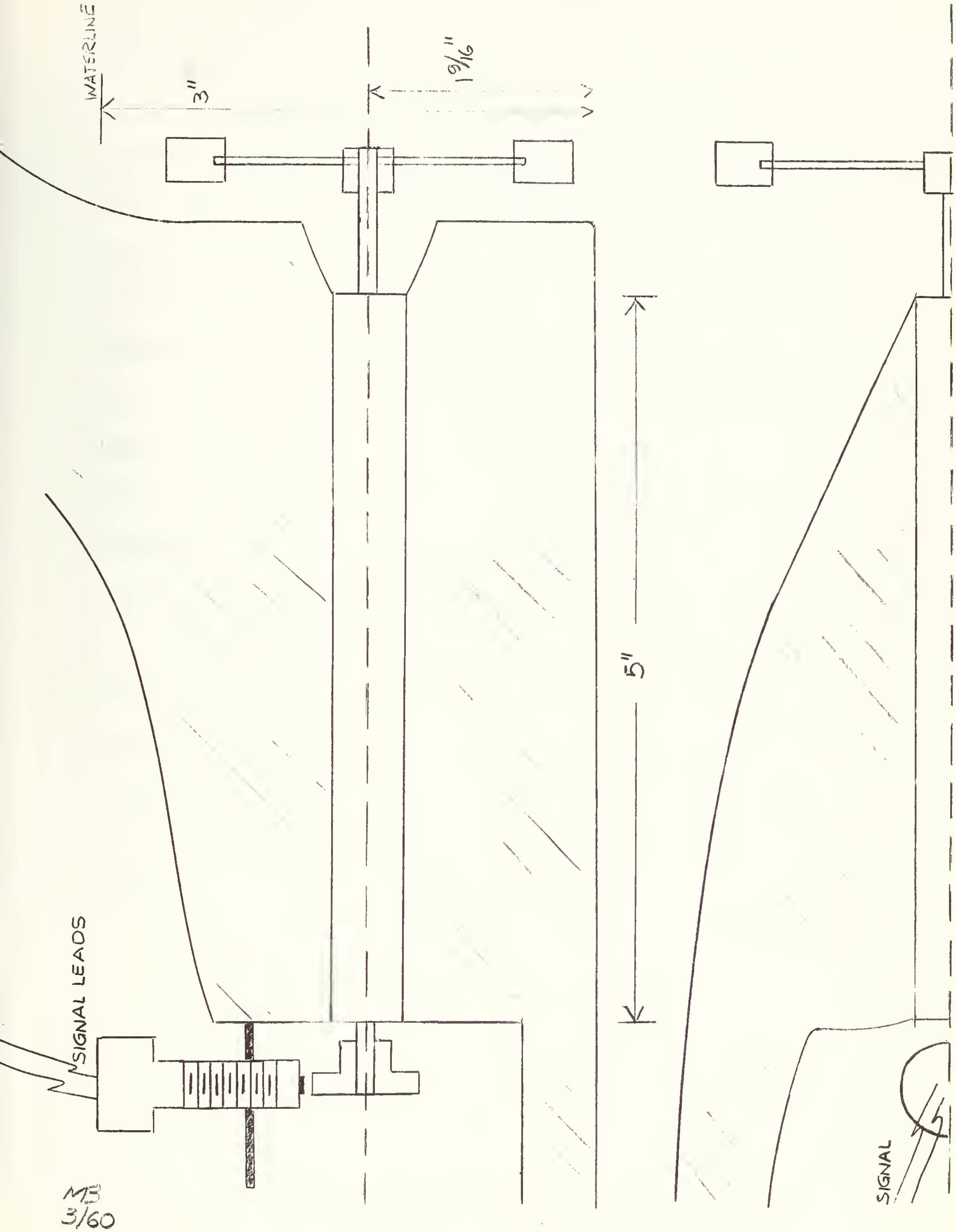


FIGURE VI

Installation of Propeller Assembly
(Scale Drawing)



the ship model to the counting unit, a one-stage transistor amplifier powered by mercury batteries was placed in the model (Figures II, III, and VII).

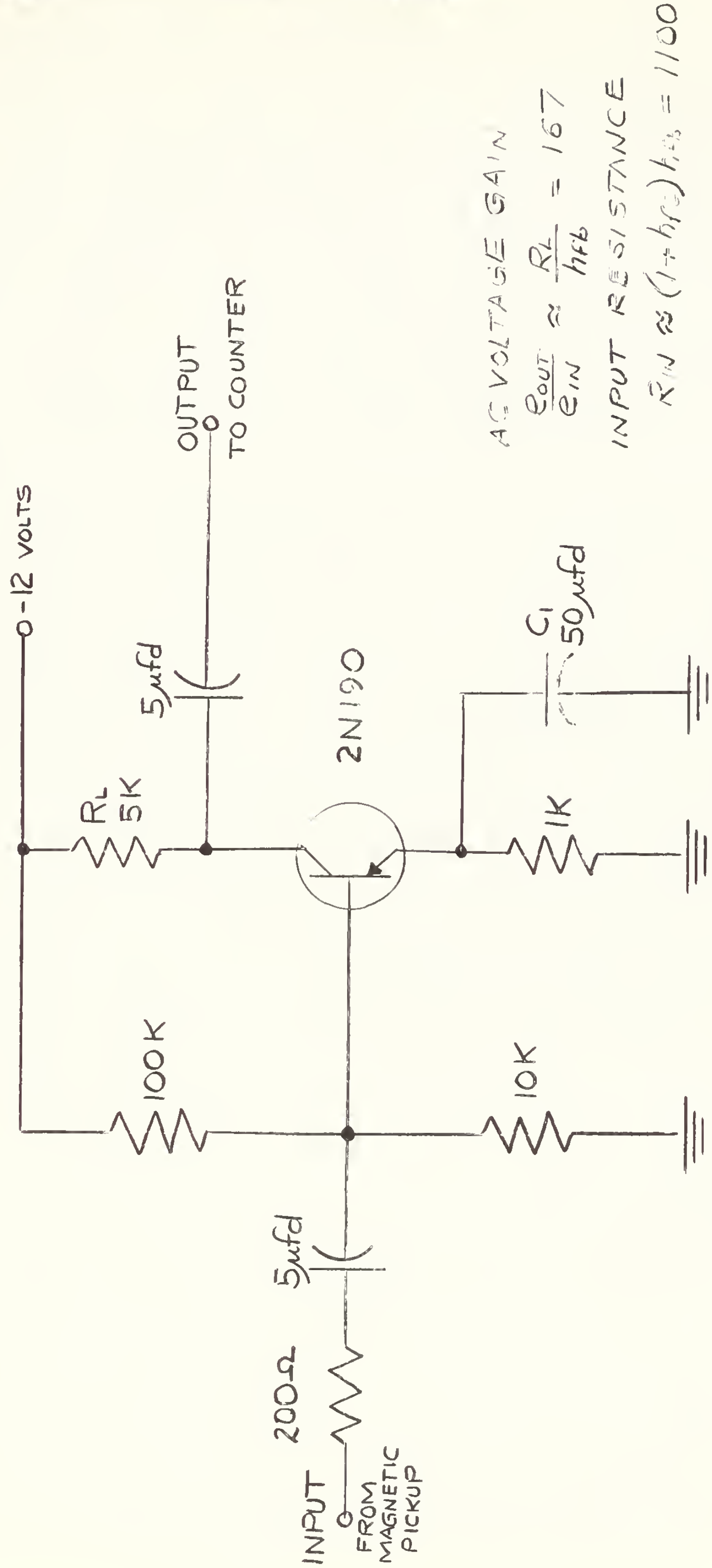
The only other problem relating to the use of a magnetic pickup was obtaining a suitable gear. It was found that small gears made of magnetic materials are not common. The gear used was a 32-pitch steel gear of 0.625 inch pitch diameter. This corresponds to a total of 20 teeth or an output signal of 20 cycles per revolution. The hub and gear face were machined to reduce the physical inertia as much as possible. This gear was found to rust very rapidly due to the moisture present from leakage through the shaft bearings, but for a short time investigation such as this the rusting was not considered to be a major problem.

One factor which adversely affects the response characteristics of the wake velocity measurement system is the friction present in the shaft bearings. Two types of bearings, journals and ball bearings were considered, but a final decision was made to use two instrument ball bearings (17) mounted as shown in Figure V.

In an attempt to provide for friction free operation, and to serve as water seals, the bearings were factory packed with a lithium soap-diester grease water resistant lubricant. This served the purpose as a lubricant but prevented any oil seepage into the towing tank. As mentioned above, the

FIGURE VII

Amplifier Circuit Diagram



MB
3/60

lubricant was also meant to serve as a water seal, but in use it was found that there was some leakage through the bearings. This leakage was not great enough to cause any problems in this investigation, but for more protracted use the shaft would have to be sealed more effectively. One possible type of shaft seal investigated for use if necessary was a mercury seal of the type used by Metzger (10).

The dynamic characteristics of the mechanical system were investigated (see Appendix C for detailed calculations) and it was found that the inertia of the vane wheel with its entrained water was so great that the inertia of the other parts of the system was negligible in comparison.

Another aspect closely related to the problem of inertia and friction is the precision of the machining required. The low friction and static and dynamic balance required calls for very close tolerances and accurate machining of the shaft-gear-vane wheel system.

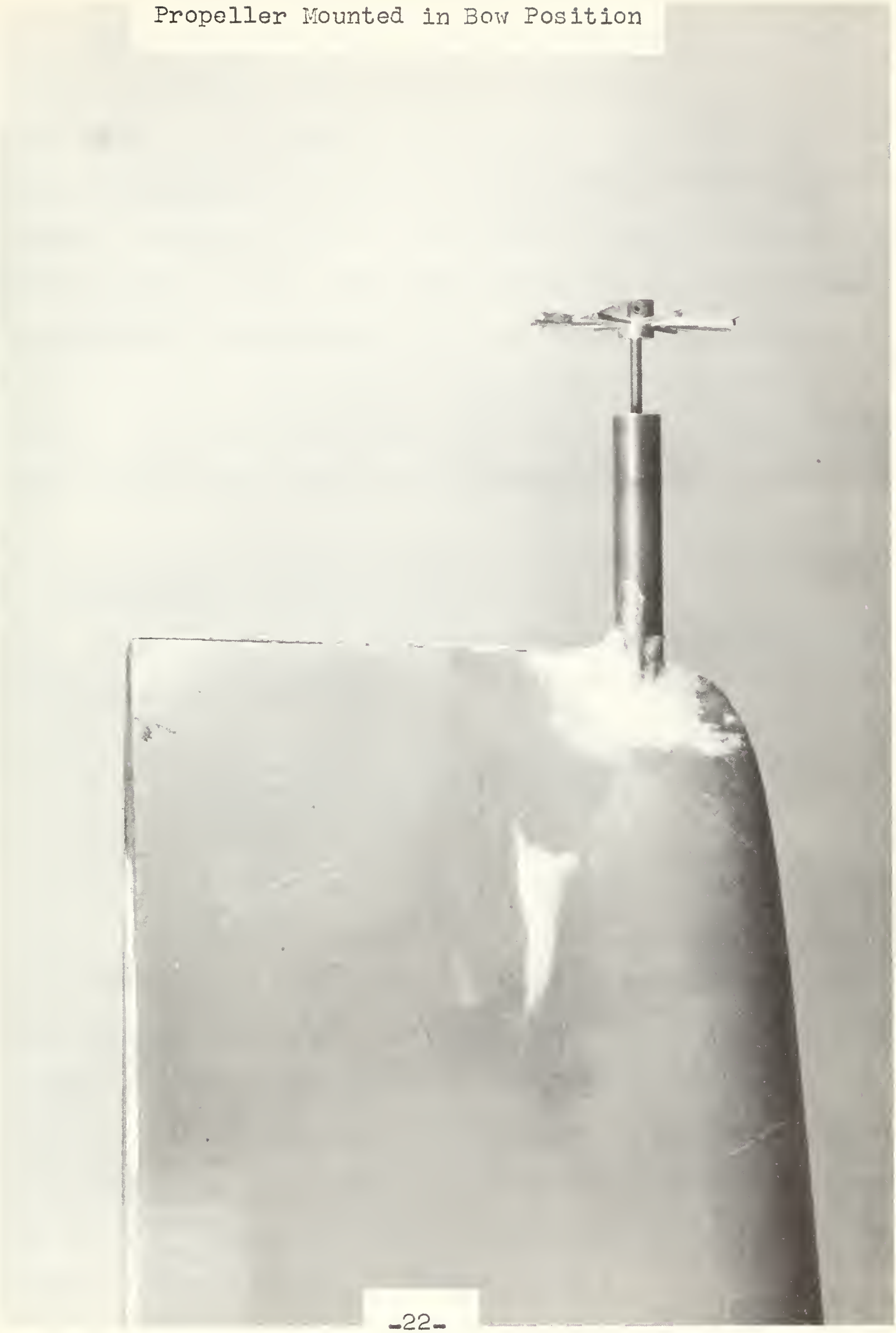
In the construction of the vane wheel it was impractical, due to the unknown friction forces present, to compute the angle at which the vane wheel should be set. Airfoil theory covering a flat airfoil (9) could be used to determine an optimum coefficient of lift. For steady state conditions the vanes should be set at such an angle so that the resultant velocity (speed of advance and rotational)

would provide zero lift. With the unknown friction resistance, it was necessary to set the angle experimentally. It was found that the shaft revolutions were quite insensitive to the blade angle used over a wide range of angles. The selected angle of 20 degrees between the plane of the blades and the plane of the propeller was an arbitrary choice, and represents a mean of the various angles investigated. The relative insensitiving of response to blade angle is explained in Appendix D.

After the vane angle had been determined and the unit fabricated, calibration runs were necessary in order to be able to convert shaft RPM to water velocity in feet per second or knots. The original calibration runs (using the Model 3055 pickup) were made with the pickup and gear enclosed in a watertight, tear-drop shaped "pod" that was suspended below and forward of the stem of the model. In this attitude, 10 inches below the keel line and five inches forward of the stern, the vane wheel was exposed to a water flow pattern that was practically undisturbed by the presence of the model's hull. This calibration, however, proved to be inaccurate because of the inability to adjust the vertical position of the pickup once the "pod" had been sealed. It was found that the signal strength was highly dependent on the clearance between the pole piece of the pickup and the gear teeth. A maximum clearance of about .005 inch was the limit for the small pickup with the relatively small angular velocity of the gear.

Figure VIII

Propeller Mounted in Bow Position



When the Model 3055 pickup was replaced by the Model 3030 the increased size of the new unit made the "pod" concept impractical. Final calibration runs were made with the vane wheel mounted in the bow of the model so that it projected $3\frac{1}{4}$ inches forward of the stem, as shown in Figure VIII. This location did not allow the vane wheel to be in an area of completely undisturbed flow, but, since variations in velocity rather than absolute magnitudes were of primary interest, the calibration was considered to be satisfactory. See Appendix A for the final calibration curve.

C. <u>Measurement of Wave Profile</u>

Water level, and therefore wave profile, can be most easily measured by either:

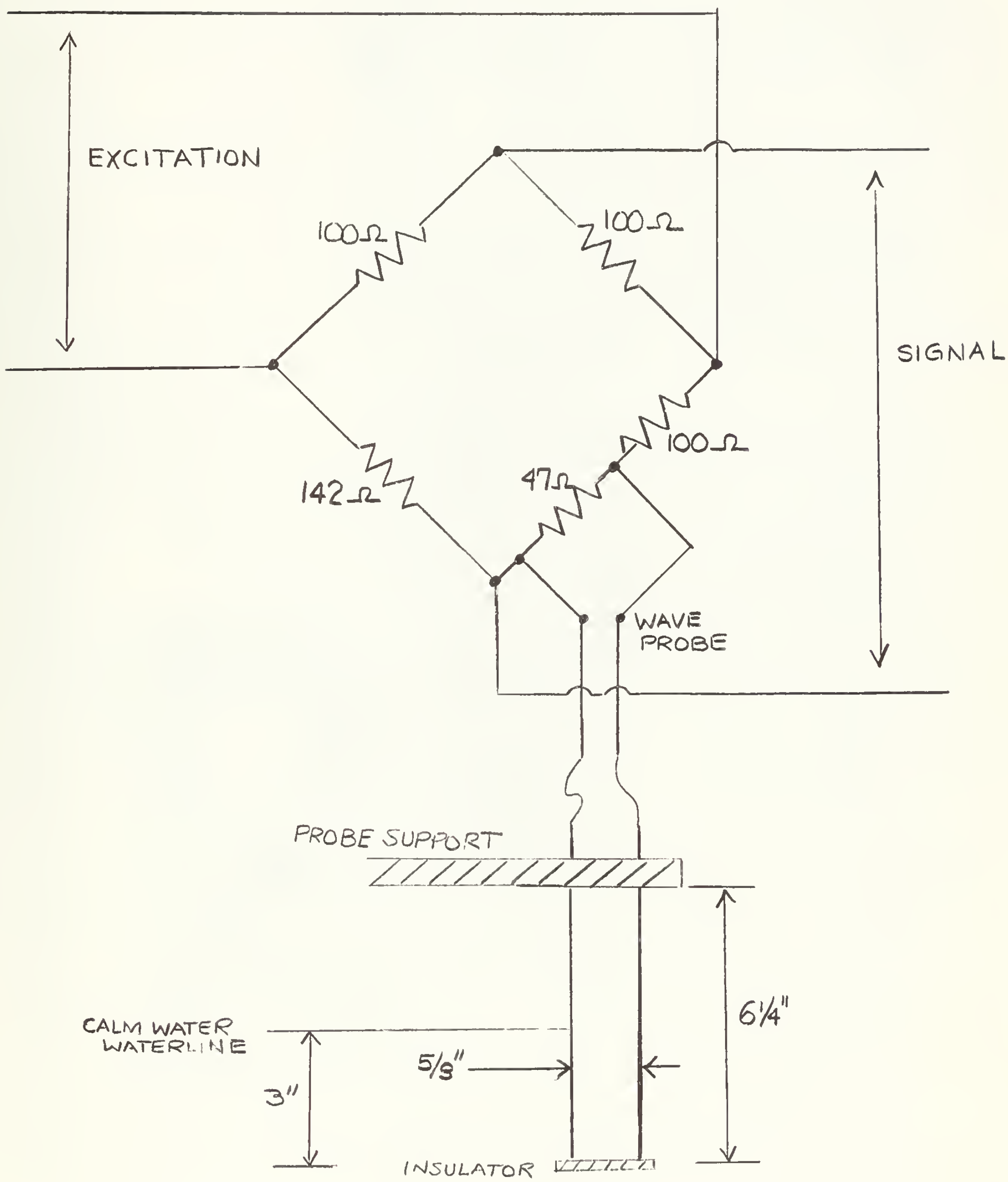
1. A capacitive bridge of the type permanently installed at the towing tank facility and described in (1), or

2. By means of a pure resistance bridge.

Because of the trouble encountered when trying to use the installed wave gage, it was decided to use a resistance bridge to measure wave profile. This device also proved to be more easily mounted on the ship model in such a manner as to eliminate flow disturbances caused by supporting members. The circuit diagram used and the measuring device itself are shown in Figures IX and X, respectively.

FIGURE IX

Wave Recorder Circuit Diagram



MB
4/60

Figure X

Wave Recorder

Using (2) as a guide, a two probe, variable resistance measuring device was developed. When calibrated for the Sanborn Recorder using the Sanborn Carrier Preamplifier as a source of excitation, the device proved to be very sensitive to changes in water level. In order to accurately determine the wave lengths and wave heights used during the investigation, the probes were held stationary and traces of the wave patterns were obtained.

D. Measurement of the Vertical Motion of the Stern of the Ship Model

Because of the availability of components, it was decided to use a linear accelerometer to measure the vertical motion at the stern of the model. The use of these instruments has been previously investigated (6), and they were found to be suitable for the measurments desired.

It was originally planned to integrate the output of the accelerometer twice to obtain a direct recording of the position of the stern. However, due to the type of excitation available from the Sanborn Carrier Preamplifier (1200 cps) it would have been necessary to demodulate, integrate, and modulate before the signal was put in the Carrier preamplifier. To avoid this complication, it was decided to investigate the possibility of using the acceleration signal directly rather than trying to convert it to position. If the acceleration trace was of a sinusoidal nature, then the

conversion from acceleration to position could be made by a simple mathematical integration. Calculations in Appendix C show that the actual acceleration closely approximates a pure sine wave. Therefore this assumption was used throughout the investigation.

A more sophisticated model motion measurement system, such as proposed by Haughey and DeLaat (6), would more precisely determine the quantities of interest, i.e., stern position and velocity. The cost of such a system, however, made its use impossible. The single vertically mounted accelerometer measured acceleration in a plane perpendicular to the baseline of the model. A small error due to pitching motion is introduced, but calculations in reference (6) show this to be negligible. Measured acceleration resulted from both heave and pitch. It would be possible with additional instrumentation to separate the two quantities, but lack of recording facilities made this refinement impractical.

When in use, the accelerometer was mounted at the stern of the model over the vane wheel as can be seen in Figure II or III.

E. Measurement of Model Speed

The permanently installed model speed measuring equipment as described in (1) was used to record model speed.

F. Test Procedure

It was originally planned to record the four desired items of information on a common time scale. However, during the period of the actual experimental runs only one frequency meter was available for use at the M.I.T. Towing Tank. Since recording of model speed and propeller RPM both required a frequency meter only one of these quantities could be measured during any one run. To overcome this difficulty, it was necessary to make each run twice under identical conditions of wave motion and model speed. For each of these two runs, a recording was made of wave profile and acceleration. Using these two traces it was possible to correlate the results of the two runs.

It will be noted in the reproductions of the recordings made (Figures B-I through B-VIII) that in some cases the wave profile is very irregular, and the crests are indistinct. This occurred when the wave length was equal to or longer than the model length. In this case it was found that the model followed the wave profile so closely that there was little relative motion between the water level and the wave probe. It would have been desirable to have the wave probe mounted in some manner so as to be unaffected by the motion of the model. If the M.I.T. Towing Tank were equipped with a carriage, the wave probe could be suspended from it, and the exact wave profile at the location of the

propeller could be recorded. Since no carriage was available, the probe was mounted on the model approximately amidships, where the only motion effect was the heave of the model.

Another source of possible error was the effect of the power and signal cables running from the model. If a carriage had been available, all the leads would have been suspended from the carriage. Without the carriage, it was necessary to run the calbes to the side of the tank and to manually, with the aid of a supporting pole, support the weight of the cables. In this situation, it was impossible to insure that the motion and speed of the model were unaffected by the cables.

Calibration of the various systems was based on the response of the Sanborn Recorder. Sensitivity was adjusted to utilize the full scale deflection of the stylus. Calibration used (mm. stylus deflection per unit input) is shown on the reproductions of the Sanborn Recordings in Appendix B.

A total of sixteen runs, covering four wave lengths, two model speeds and various wave heights were made to obtain the results included in Tables I-IV.

III.

RESULTS

As stated in the Introduction, it was hoped that these tests show clearly and conclusively that the wake variation was directly connected, both in magnitude and phase, to the orbital velocities of the waves encountered by the model. For various reasons, which will be explored more fully later, the desired results were not obtained. While it appears that orbital velocity is a major contributor to the wake variation, there are strong indications that other effects exist which may be of equal importance. Much time was devoted to an attempt to separate these various effects. Their significance in the indicated wake variation shown in these results will be covered fully in the Discussion of Results.

A summary of all the data obtained from the sixteen model runs is listed in Tables I-IV. No attempt has been made in this tabulation to account for errors or phase lags in any part of any of the systems; the results are presented exactly as they were obtained during the course of the tests. One simple shift in time scale was necessary

in order to compensate for the distance between the wave probe (located 2.22 feet forward of the propeller) and the actual propeller location. The details of this calculation are shown in Appendix C (Figure C-II).

To illustrate clearly the random behavior of the wake variation, sketches of the magnitude-time relationships of the variables are presented as Figures XI-XVIII. The time scale of these plots is based on the period of encounter (T_e) of the model with the waves. These figures are intended to show the relationship at any given time between the actual height of the wave at the location of the propeller and the motion of the stern (i.e. the centerline of the propeller). The time of the maximum wake velocity, determined from the Sanborn recordings, is indicated by the single vertical line. Phase lags and leads shown are based on 0° at the wave crest.

Figures XI-XVIII represent only half of the runs made, and include representative runs from all four wave lengths investigated. The actual recordings from which these results were obtained are reproduced and included in Appendix B.

It should be noted that in the case of the two long wave lengths (Figures XI-XIV) the sketches show that the propeller was out of the water, at least partially, when the stern was at its highest position. Visual observation

confirmed that the propeller was breaking water during runs 31-34. Examination of the actual Sanborn recordings shows that the wake trace (CPS) is uneven and jagged at this time (near the minimum CPS), which is further confirmation that the propeller was in fact breaking the surface of the water.

There is also the possibility that the assumption of sinusoidal acceleration, and the analytical integration to obtain position, introduced a peak magnitude error in the position results. This error, if present, is not considered significant.

The period of encounter for all runs was determined from the Sanborn recordings, using an arithmetic average over the steady portion of the run. This period was computed separately for each of the variables, and the results compared. Wake period,acceleration period, and relative period of encounter of the waves in all cases agreed to within 2%, a clear indication that the magnitude variations were in some manner related. There was no similar relationship that included the indicated variations in model speed. The large speed variations shown for some of the runs, attributed to resonance in the towing cable, are covered in the Discussion of Results.

These results, which show no clear relationship between wake variation and any of the other quantities measured,

demand much explanation and clarification. For this reason, the remaining experimental time was devoted to ascertaining the reasons and significance of the various causes and effects.

TABLE I

Summary of Results

Runs	19-20	21-22	23-24	25-26
Wave Length (ft)	9.06	9.06	8.85	8.85
Wave Height (in)	2.5	2.5	1.6	1.6
Maximum Orbital Velocity (ft/sec)	0.487	0.487	0.316	0.316
Average Model Speed (kts)	1.80	1.40	1.55	1.85
(ft/sec)	3.04	2.36	2.62	3.12
Period of Encounter (sec)	0.934	0.980	0.940	0.900
Frequency of Encounter	1.07	1.02	1.06	1.11
Average Wake Velocity (kts)	1.19	0.95	1.10	1.30
(ft/sec)	2.00	1.60	1.86	2.20
Wake Fraction	0.342	0.322	0.290	0.295
Wake Variation (± kts)	0.210	0.210	0.279	0.316
(± ft/sec)	0.356	0.356	0.472	0.534
Phase (wake to wave crest at propeller)	-202°	-198°	-201°	-206°
Maximum Stern Acceleration (± G's)	0.35	0.265	0.225	0.250
Maximum Stern Motion (± in)	3.00	2.49	1.95	2.00
Phase (stern up to wave crest at propeller)	-22°	-18°	-21°	-26°

TABLE II

Summary of Results

Runs	27-28	29-30	31-32	33-34
Wave Length (ft)	7.13	7.13	7.10	7.10
Wave Height (in)	1.2	1.2	1.4	1.4
Maximum Orbital Velocity (ft/sec)	0.264	0.264	0.308	0.308
Average Model Speed (kts)	1.60	1.35	1.00	1.15
(ft/sec)	2.70	2.38	1.69	1.94
Period of Encounter (sec)	0.800	0.850	0.910	0.880
Frequency of Encounter	1.25	1.17	1.10	1.13
Average Wake Velocity (kts)	1.125	1.02	0.622	0.685
(ft/sec)	1.90	1.72	1.05	1.16
Wake Fraction	0.296	0.277	0.379	0.402
Wake Variation (± kts)	0.221	0.252	0.263	0.211
(± ft/sec)	0.373	0.426	0.444	0.356
Phase (wake to wave crest at propeller)	-176°	-180°	-184°	-182°
Maximum Stern Acceleration (± G's)	0.30	0.35	0.45	0.50
Maximum Stern Motion (± in)	1.88	2.47	3.65	3.79
Phase (stern up to wave crest at propeller)	+22°	+17°	+12°	+14°

TABLE III

Summary of Results

Runs	35-36	37-38	39-40	41-42
Wave Length (ft)	5.05	5.05	5.12	5.12
Wave Height (in)	2.2	2.2	1.3	1.3
Maximum Orbital Velocity (ft/sec)	0.574	0.574	0.337	0.337
Average Model Speed (kts)	1.35	0.95	1.55	1.95
(ft/sec)	2.28	1.60	2.61	3.29
Period of Encounter (sec)	0.694	0.740	0.694	0.640
Frequency of Encounter	1.44	1.35	1.44	1.56
Average Wake Velocity (kts)	0.87	0.728	0.93	0.80
(ft/sec)	1.47	1.23	1.57	1.35
Wake Fraction	0.355	0.231	0.390	0.590
Wake Variation (± kts)	0.074	0.095	0.084	0.063
(± ft/sec)	0.124	0.160	0.142	0.106
Phase (wake to wave crest at propeller)	-206°	-183°	-267°	-272°
Maximum Stern Acceleration (± G's)	0.35	0.325	0.375	0.22
Maximum Stern Motion (± in)	1.66	1.75	1.51	0.88
Phase (stern up to wave crest at propeller)	+18°	+16°	-45°	-47°

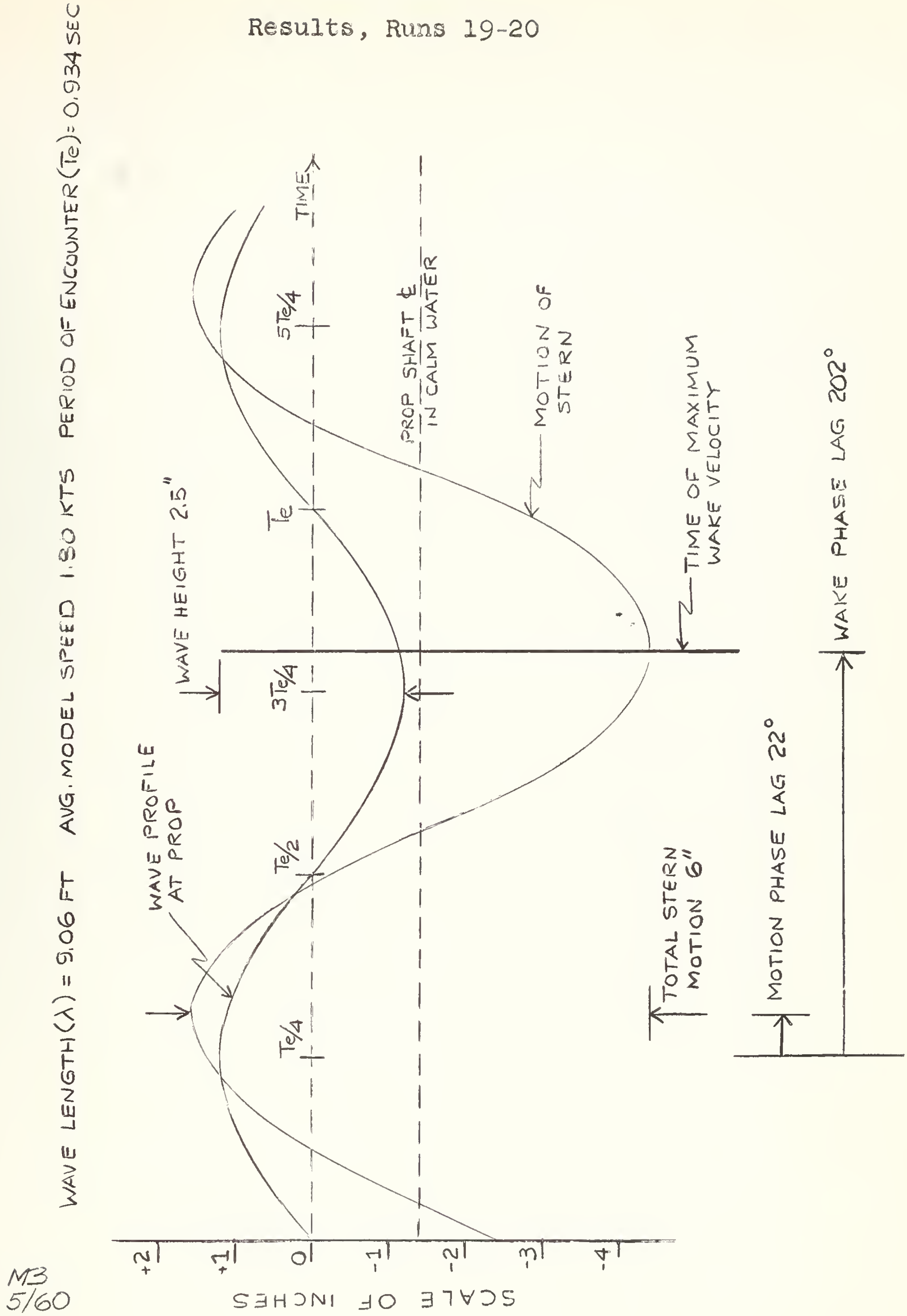
TABLE IV

Summary of Results

Runs	43-44	45-46	47-48	49-50
Wave Length (ft)	3.18	3.18	3.15	3.15
Wave Height (in)	1.4	1.4	1.9	1.9
Maximum Orbital Velocity (ft/sec)	0.461	0.461	0.630	0.630
Average Model Speed (kts)	2.07	1.85	1.65	1.90
(ft/sec)	3.49	3.12	2.78	3.21
Period of Encounter (sec)	0.420	0.440	0.467	0.440
Frequency of Encounter	2.38	2.28	2.14	2.28
Average Wake Velocity (kts)	1.15	1.04	1.02	1.19
(ft/sec)	1.94	1.76	1.72	2.01
Wake Fraction	0.443	0.436	0.381	0.374
Wake Variation (± kts)	0.042	0.032	0.053	0.063
(ft/sec)	0.071	0.053	0.089	0.106
Phase (wake to wave crest at propeller)	-109°	-109°	-106°	-106°
Maximum Stern Acceleration (± G's)	0.03	0.05	0.07	0.07
Maximum Stern Motion (± in)	0.05	0.08	0.15	0.15
Phase (stern up to wave crest at propeller)	-109°	-109°	-106°	-106°

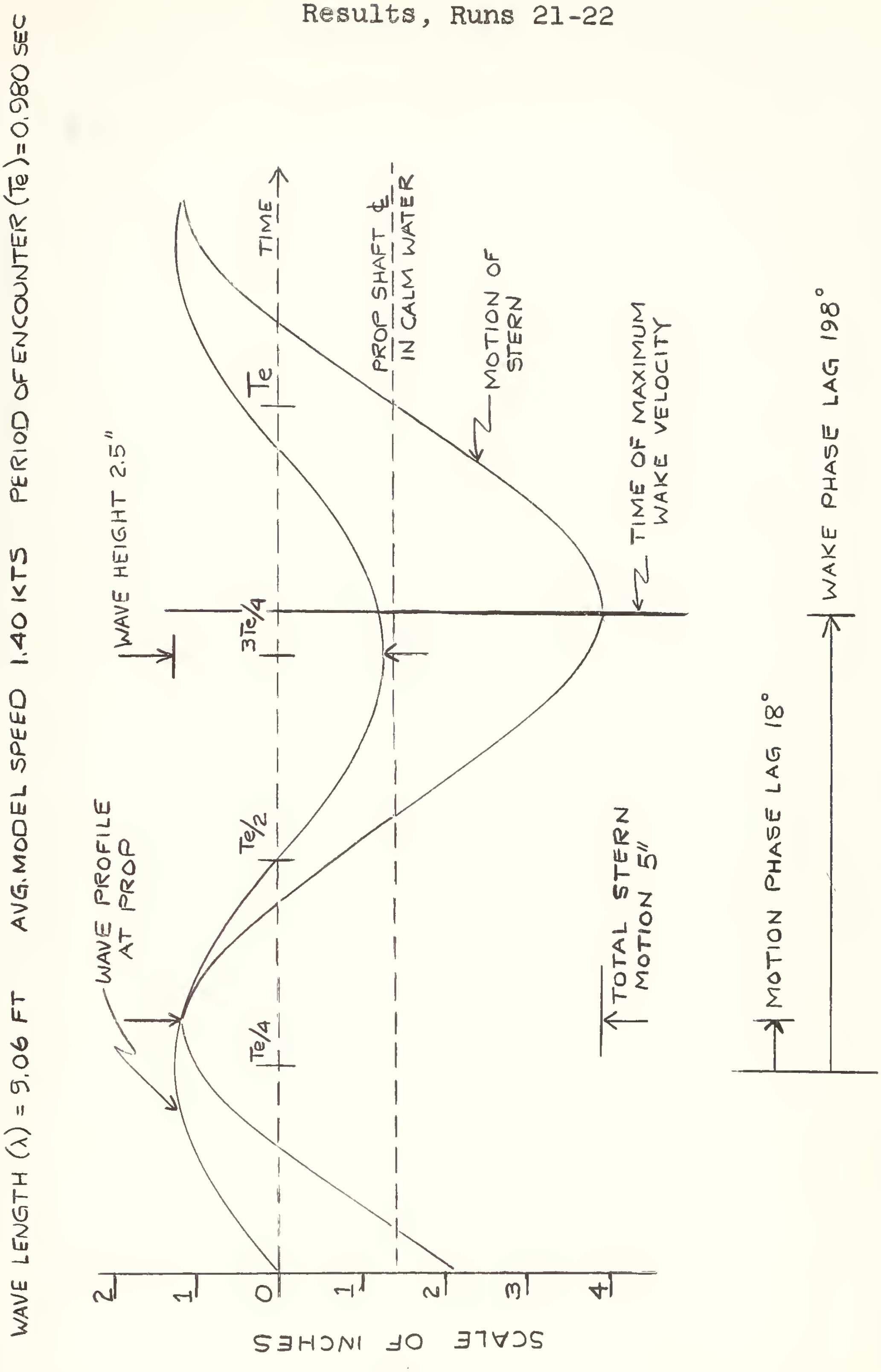
FIGURE XI

Results, Runs 19-20



<u>FIGURE XII</u>

Results, Runs 21-22



MB
5160

FIGURE XIII

Results, Runs 31-32

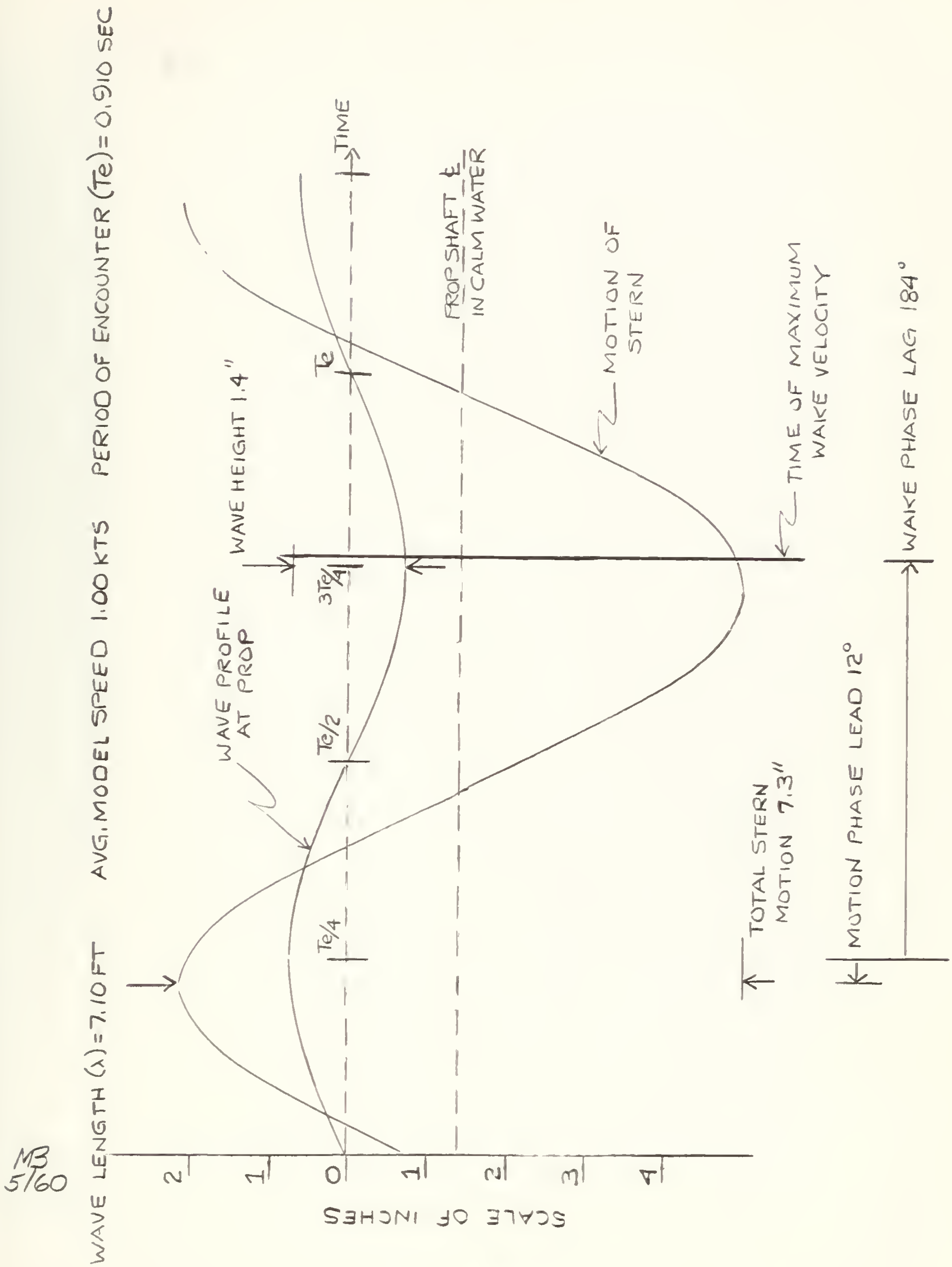


FIGURE XIV

Results, Runs 33-34

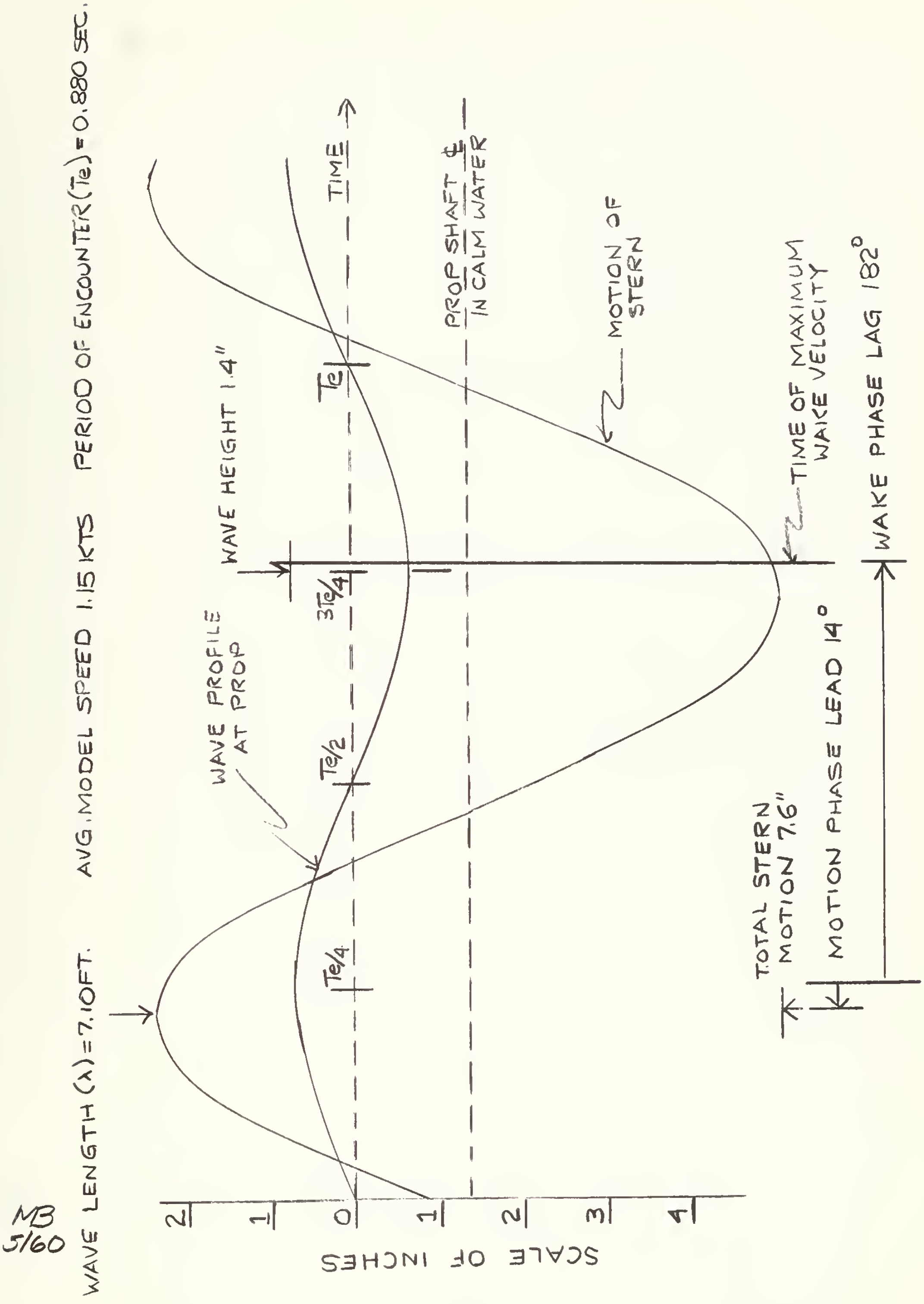


FIGURE XV

Results, Runs 39-40

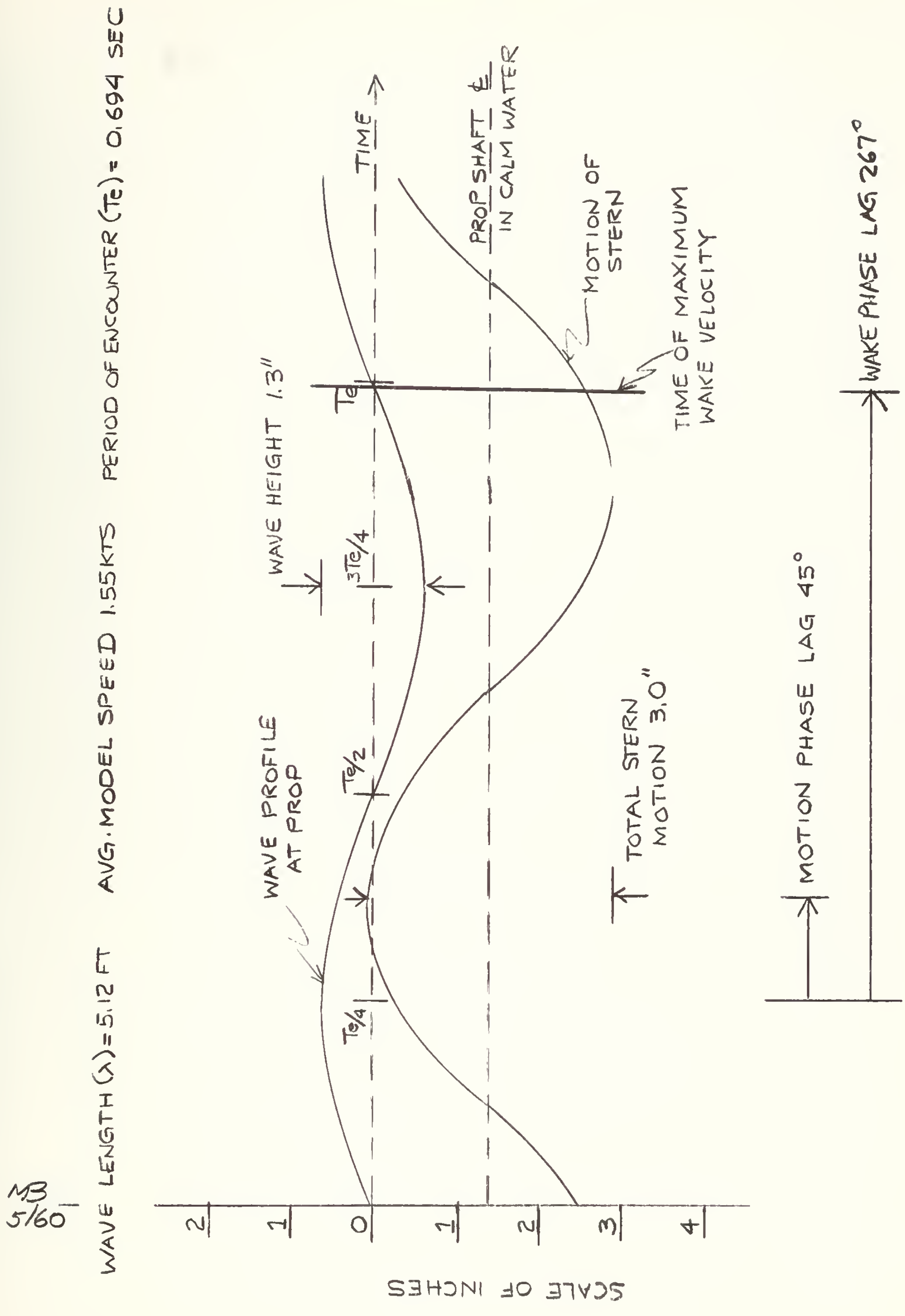


FIGURE XVI

Results, Runs 41-42

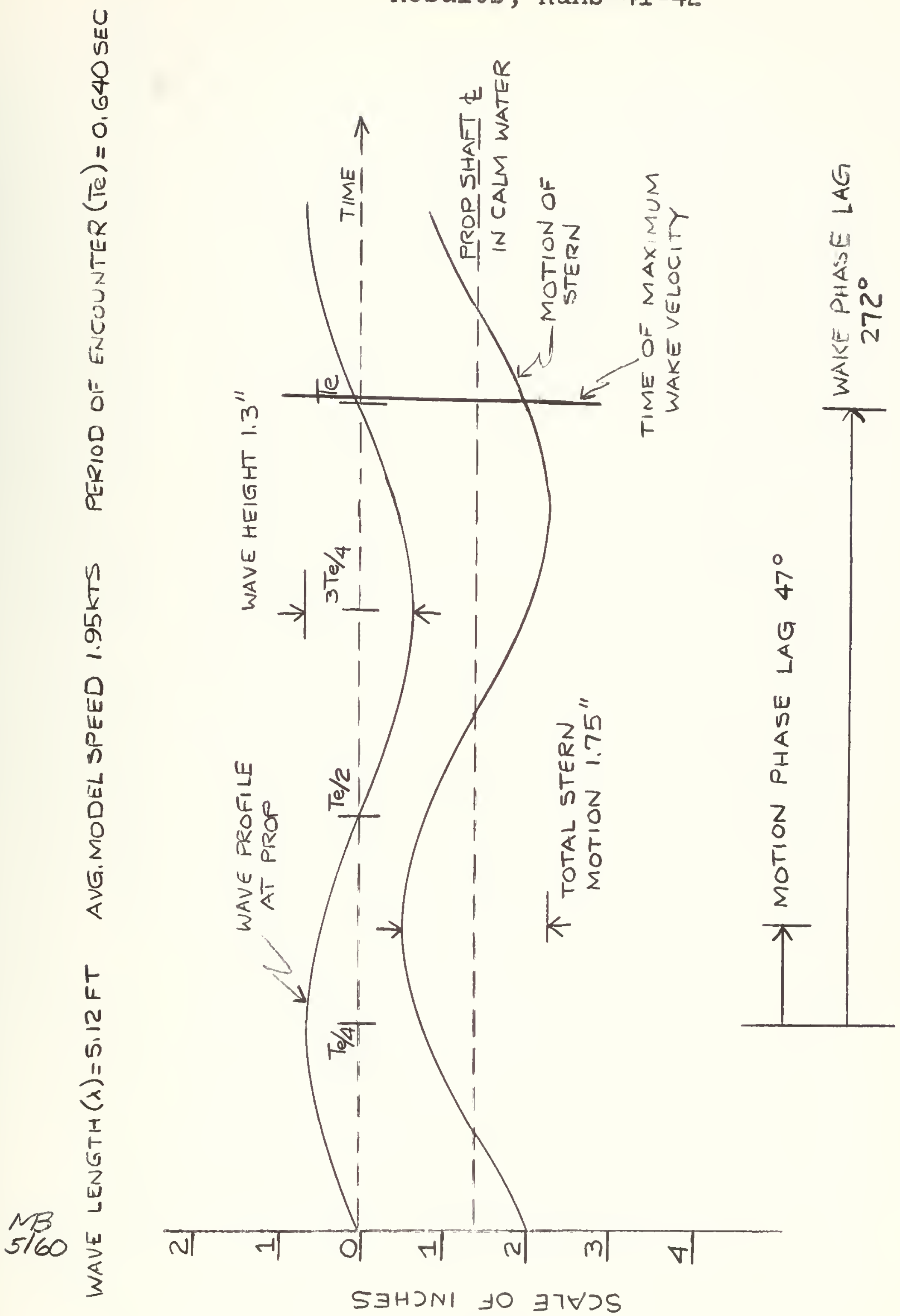


FIGURE XVII

Results, Runs 47-48

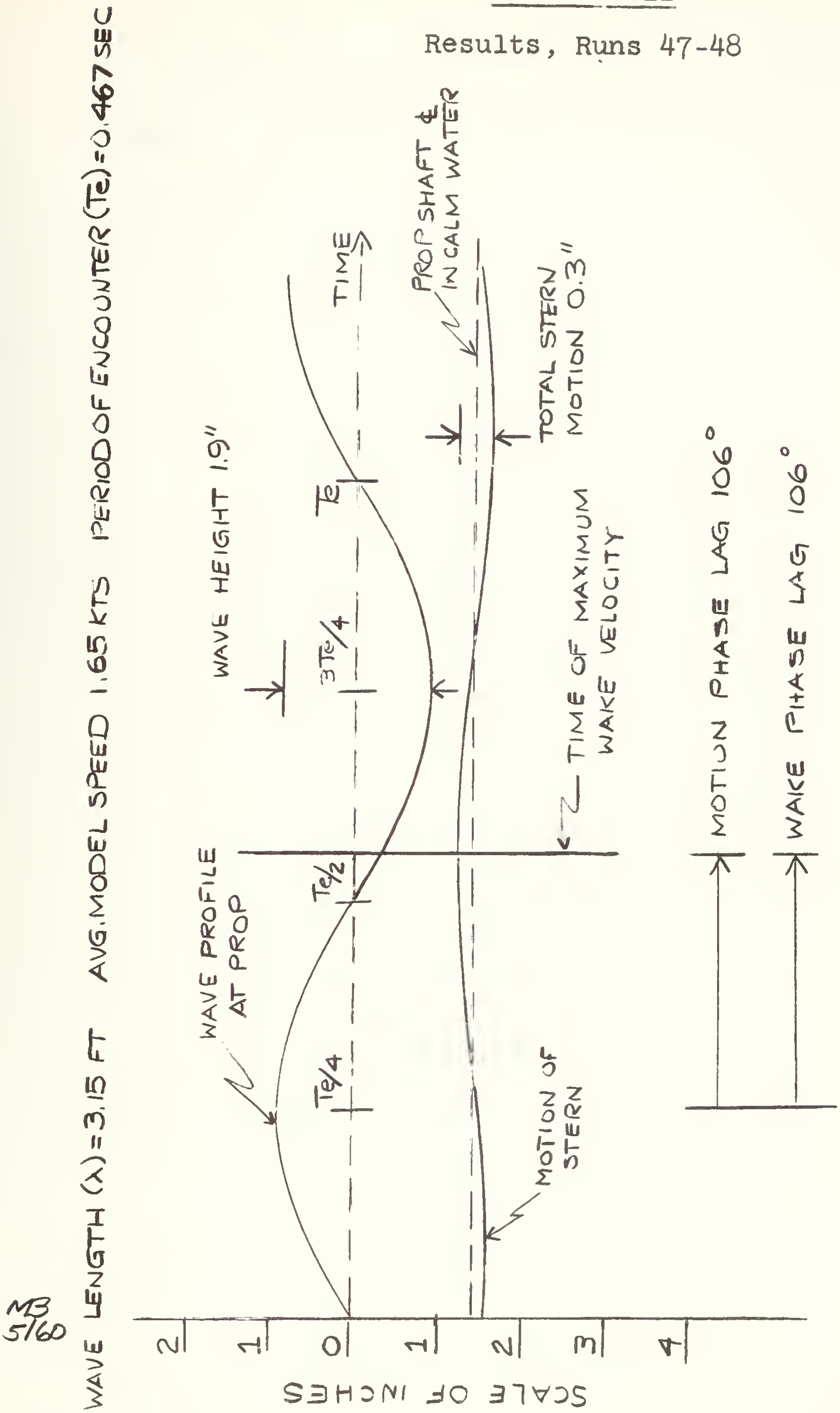
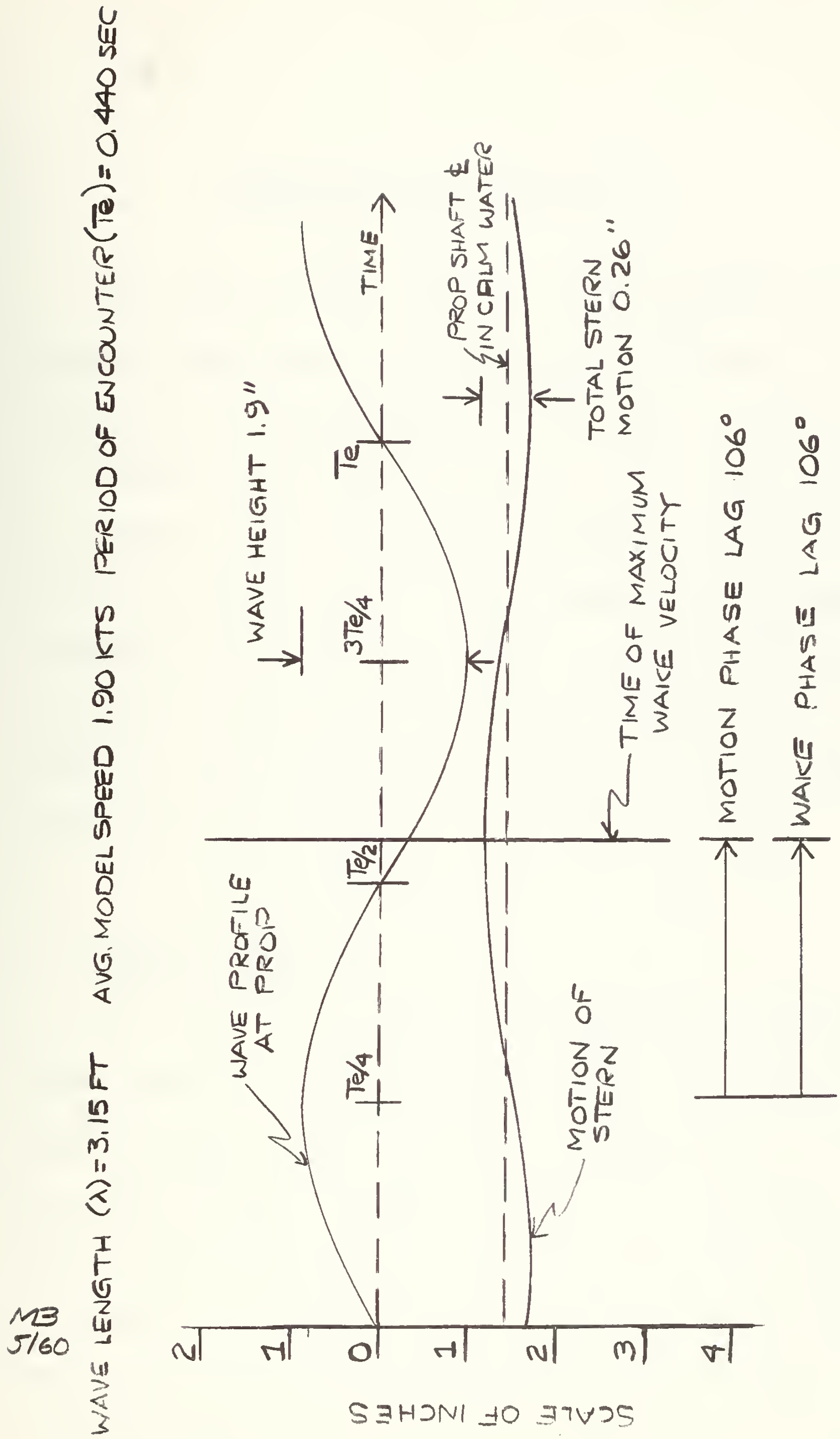


FIGURE XVIII

Results, Runs 49-50



IV.

DISCUSSION OF RESULTS

The results obtained during the sixteen model runs provide more questions than they do answers. The assumption has been made that wake variation in regular waves is caused by the orbital velocity of these waves. If this is correct, then the maximum point of the wake (CPS) trace should coincide with the wave crest on a common time scale. Figures XI-XVIII do not show any such relationship. The discussion to follow is intended to explore as fully as possible the reasons for the wide variance between theory and actual test results.

A. Speed Variation

The first apparently erroneous result to be investigated was the apparently large variation in model speed that occurred during some of the model runs. This will be particularly noticed in Runs 47-48 (Figure B-VII), where the peak to peak speed variation indicated is about 0.75 knots. Previous observations at the M.I.T. Towing Tank have shown that speed variations of models towed in regular waves is negligible. Careful calculation and com-

parison of the periods of these speed oscillations in all runs showed all periods to be integer multiples of about 0.46 seconds. It was also noted that during runs where the period of encounter was near 0.46 seconds, or an integer multiple of 0.46 seconds, the magnitude of the indicated variation was very large. At other periods the magnitude was much less. This led to the conclusion that the resonant period of the towing mechanism-speed recording device was about 0.46 seconds.

To check this conclusion, two further tests were made. The tension in the towing cable was varied, and the model towed in regular waves of varying wave lengths. Increasing the tension in the towing cable caused the apparent resonant period to decrease; decreasing the tension caused a corresponding increase in period.

As a final check that the indicated speed variation was in fact occurring in the 100 feet long string, and not in the model, the accelerometer used for the stern acceleration measurements was remounted amidships in a horizontal position, in such a manner to indicate model surge. While it was impossible because of other equipment already in the model to mount the accelerometer in such a manner to completely eliminate the effects of pitching, the test did serve to give an indication of the horizontal acceleration experienced by the model. This

test was conducted under the same wave conditions as Runs 47-48, in which the maximum speed variation of the model had apparently occurred. Integration of the horizontal acceleration trace recorded during these runs showed a maximum model speed variation of less than one tenth that shown by the speed recording equipment

As a result of these investigations, variations in model speed indicated during the regular test runs were determined to be false readings, caused by resonance in the towing apparatus. The remaining portion of the analysis of the results is based on the assumption that any model speed variation that does exist is negligible, and has little or no effect on the variation of wake. Recordings of model speed were used only to obtain the average model speed during any run.

B. Other Sources of Error

Once model speed had been eliminated as a possible source of error, the other three variables were examined more closely. The most striking thing about the results from all the runs was that the recorded peak wake occurred at least 90° after the wave crest was at the propeller, and in many cases it lagged 180° or more. This result led to several investigations.

1. Possible phase inversion of one or more of the variables. Particularly in the case of the two longer wave lengths investigated, phase inversion of either the wave or wake trace would bring the two approximately into the theorized relationship.

2. Phase lags in one or more of the systems. It was decided to closely check the frequency response characteristics of the combined mechanical-electrical propeller speed recording system.

3. Wake variation caused by other effects. It may be noted from the results that in all but the shortest wave lengths (when the vertical motion of the stern was very small), the peak of the wake trace occurred near the time when the stern was at its lowest position. If this were an accurate result, then it would be reasonable to assume that there was a definite effect on the wake velocity from either the position or velocity of the stern, or possibly both.

To follow are the results of the investigations into these possible sources of error, and a discussion of the effect these errors would have on the accuracy of the results.

C. Phase Inversion

180° phase reversal would be a possibility in any of the three systems if errors had been made in the original analysis or in the connection of the apparatus. Therefore, each system was checked separately.

The mechanism for recording the wake velocity is a simple electronic counting system. Voltage pulses are generated in the magnetic pickup by a variable reluctance principle each time one of the gear teeth passes close to the pole piece of the pickup. The use of the single-stage amplifier does reverse the phase in the sense that it makes the positive pulses negative. Since both the counter and the frequency meter react to a positive slope of the input voltage, and produce a DC level signal proportional to the number of positive slopes detected in a given counting period, no time lag or phase reversal in the output signal to the Recorder is possible.

This reasoning was checked by bypassing the amplifier in the counting circuit. At high speeds the signal from the pickup was great enough to produce a recordable signal. These tests confirmed the fact that a complete 180° phase shift was impossible in the counting circuit.

Phase reversal in either the wave or acceleration measurement circuits would be more of a possibility. Both measure a difference from a steady state value. In the case

of the acceleration, the zero value is the normal earth's gravitational force. The wave probe was zeroed at the calm water waterline. To check the accuracy of these recordings, a series of tests, largely based on visual observations, was made. These tests were designed to indicate positively the direction of stylus deflection for various inputs, and to insure that the proper relationship between wave position and acceleration was in fact being recorded. The details of these tests, and the results obtained are covered fully in Appendix D.

As a result of this investigation, it was shown beyond all doubt that the recordings of wave position and acceleration were correct both as to polarity and magnitude, and that there was no possibility of erroneous results from a phase reversal in these systems.

D. Phase Lag

Even if there were no complete 180° phase shifts, it would still be possible for one or more of the systems to have a frequency response characteristic that would cause a phase lag in the recorded results. The checks made for phase inversion in the wave and acceleration circuits eliminated any chance of phase lag to any significant degree in these circuits. Even the possibility that capacitance in the long cable from model to instrument

location might have some effect was checked. Effect of line capacitance would be small because of the low frequencies of the variations. It can also be assumed that all the signals would be subject to the same attenuation, and that the proper phase relationship between signals would be maintained.

The wake measurement circuit presents more possibilities for time lag. This is a combined mechanical-electrical system, and both mechanical and electrical time constants are of importance. Electrically there was little possibility for any phase lag, and this was confirmed by introducing a step change of frequency into the counter, and recording the time response. While this was a very crude way of checking frequency response, it did serve to indicate that the electrical time constant was at least an order of magnitude less than that which would be necessary to account for the phase lags shown in the results.

The large unknown in the overall frequency response is the relationship between the wake velocity and the torque applied to the rotating propeller. Also unknown is the resistance torque of the bearings and of the rotating propeller in water. These unknowns were therefore the subject of further investigation.

Time and lack of proper instrumentation prevented an exact determination of the many variables that may affect the propeller response. By a series of tests, it was possible to approximate many of the relationships involved. These tests, and the results obtained, are shown in detail in Appendix D.

The results of these tests confirmed the original assumption that the inertia and friction effects of the shaft assembly itself would be very small compared with the effect of entrained water. The results also give a strong indication that the propeller-wake measuring system was being operated at or near the high frequency cut-off where an appreciable phase lag should be expected. This cut-off is primarily caused by the entrained water, and its effect on both the inertia of the system, and the resistance torque proportional to rotational speed.

The conclusion that must be reached at this point is that for further investigations into wake variation, very careful attention must be paid to the propeller assembly in order to insure that it will operate in a portion of the frequency response curve where phase shift is not present, or if this proves impractical it must be operated where the phase lag is constant over the frequencies encountered.

E. Other Effects

While the frequency response characteristics of the shaft system were apparently extremely important in affecting the results, the total phase lag present in these results cannot be completely explained by this cause. It was for this reason that an effort was made to determine if there was another cause of wake variation.

The other variable measured, which has not been considered to this point, was the acceleration (position) of the stern. Even though the propeller itself is insensitive to velocities in a plane parallel to the plane of the propeller, the effect of stern motion on wake variation was investigated. This was prompted by two observations. First was the fact that the results (Figures XI-XVIII) show that the peak of the wake velocity recording tended to follow the motion of the stern. Secondly, it was observed that even in calm water a manually induced pitching and/or heaving motion of the model caused a definite variation in the propeller speed. This model motion was very irregular, since it could only be induced by the cable support pole, but it did serve to act as a starting point for further investigation.

The results of varying the trim and displacement of the model in calm water are shown in Appendix E. They show that trimming the model about its normal waterline did have

an appreciable effect on the steady state propeller speed. They further showed that the greatest propeller speed, and therefore the greatest wake velocity, occurred when the model was trimmed by the stern. This condition generally occurred when the wave trough was at or near the propeller. The resulting effect on wake velocity would be exactly opposite to that from the orbital velocity of the waves.

F. Correction of Results

Having determined that there was possibility of error in the results and also that wake variation might be caused by effects other than orbital velocity, the next step was to apply corrections to the original results in an attempt to show approximate correct relationships. Correcting experimental results with other experimental data is at best a hazardous procedure, and it is particularly so when the accuracy of the corrections is subject to doubt. Such a procedure is of value primarily because it serves to indicate that all possible errors have been accounted for, and warns future workers in this field where and of what order of magnitude errors are likely to occur.

Two separate runs were selected to apply the corrections to. Runs 31-32 represent a relatively long (7.10 feet) wave length, and runs 47-48 represent a short (3.15 feet) wave

length where model motion is small. The correction procedure, and the modified results are contained in Appendix F.

V.

Conclusions

The experimental work conducted had four principal objectives. The objectives were based on determining the answers to four questions, contained in the Introduction. As a result of the analysis of results, the following conclusions may be reached:

(1) The magnitude of the wake velocity definitely does vary for a model operated in regular ahead seas. This variation is of sufficient magnitude that it can be measured in a small Ship Model Tank facility.

(2) The orbital velocity of the water particles in the waves is a definite contributor to wake variation. Variation in waves of very short wave lengths in relation to ship length is very nearly in phase with, and of corresponding magnitude to these orbital velocities.

(3) Model motion has a definite effect on wake variation. For wave lengths that produce large pitch angles, the phase and magnitude of wake variation cannot be determined by considering orbital velocity theory alone.

(4) Prediction of full scale effects is not possible until the two causes of wake variation, orbital velocity and motion, are analyzed more thoroughly and their interdependence determined.

While the results obtained do not prove conclusively any theory as to wake variation, they do show a method for further work in this field. More exact results are possible by the use of more sophisticated instrumentation, and if care is taken to avoid or eliminate the errors found in this analysis.

The magnitude of wake variation indicated by these experimental results is sufficient to affect the propeller operating parameters. To fully analyze the performance of a propeller under all operating conditions, it is necessary that wake variation from all causes be accurately known.

VI.

Recommendations

A. Equipment Modifications

One major problem encountered in this investigation was the difficulty in obtaining an undisturbed picture of the wave profile at the stern of the model. The only way to avoid this problem is to conduct any further investigations of this type at a facility equipped with a carriage to which the wave measuring device could be attached. It is proposed to install such a carriage at the M.I.T. Towing Tank, so this problem could easily be solved in any further work.

As discussed in Appendix D, a lower propeller rotational speed is indicated in order to reduce the time constant in the system. This however presents a problem in measuring the rotational velocity. The sensitivity of a magnetic pick-up decreases with a decrease in speed, and replacement of this measuring system with a more sensitive device, such as the proximity transducer discussed in the Procedure, should be seriously considered.

In order to more effectively evaluate the effects of model motion, it is felt that a minimum of two accelerometers should be used. This however, would require more recording

facilities than are now available. A gyroscopic type of angular measurement system could measure the pitching motion alone if the extra recording channel was not available. Another alternative would be to devise some type of measurement system that could be attached to the supporting carriage and would measure the difference in vertical displacement between the bow and the stern of the model. This system would also require only one recording channel.

As discussed in Appendix D, there is an inherent phase lag in the propeller type wake measuring system. This lag can be partially controlled by judicious choice of parameters, but it can be eliminated only by the use of an alternate type of measuring system, such as the thermistors discussed in the Procedure.

The final equipment recommendation is to make any further investigations at a facility equipped with at least four Sanborn Recorders and associated amplifiers. This will eliminate the necessity of making dual runs and correlating the results.

B. Procedural Modifications

It was found in this investigation that the variation in wake velocity is probably caused by two separate and distinct effects -- orbital velocity and model attitude

(Appendix E). It is therefore recommended that the two causes and their effects be studied independently. This could be done by several methods:

(1) Inducing a pitching motion in the model by some externally controlled means, such as a cam and rod mounted on the carriage, and then measuring the wake variation in calm water.

(2) Mounting the vane wheel assembly in a propeller tunnel in such a manner that the angle between the shaft and the oncoming flow could be varied.

(3) Securing the model to the towing carriage in such a manner that it could not pitch and then measuring the wake variations that occur at the propeller location.

VII.

APPENDIX

APPENDIX A

Calibration of Equipment

All calibration was based on the response of the Sanborn Recorder. Each system was calibrated so a given incremental change in the variable would cause an equivalent displacement of the recorder stylus. The final calibration scale used may be seen on the reproduction of recordings in Appendix B.

The only system that required any special calibration was the propeller system. The signal generated by the pickup was of variable frequency proportional to the speed of the gear teeth past the pole piece. The Recorder was calibrated to read in cycles per second. Twenty teeth on the gear meant one revolution per second was equivalent to 20 cycles per second.

To equate the propeller speed with the relative water velocity, a series of calibrating runs with the propeller located in the bow (Figure VII) was made. The results of this calibration are shown in Figure A-I. This curve allows conversion of the propeller signal from CPS to knots. The faired line was computed to have a slope of 95 cycles per knot, and this relationship was used in computing the results.

FIGURE A-I

Propeller Calibration

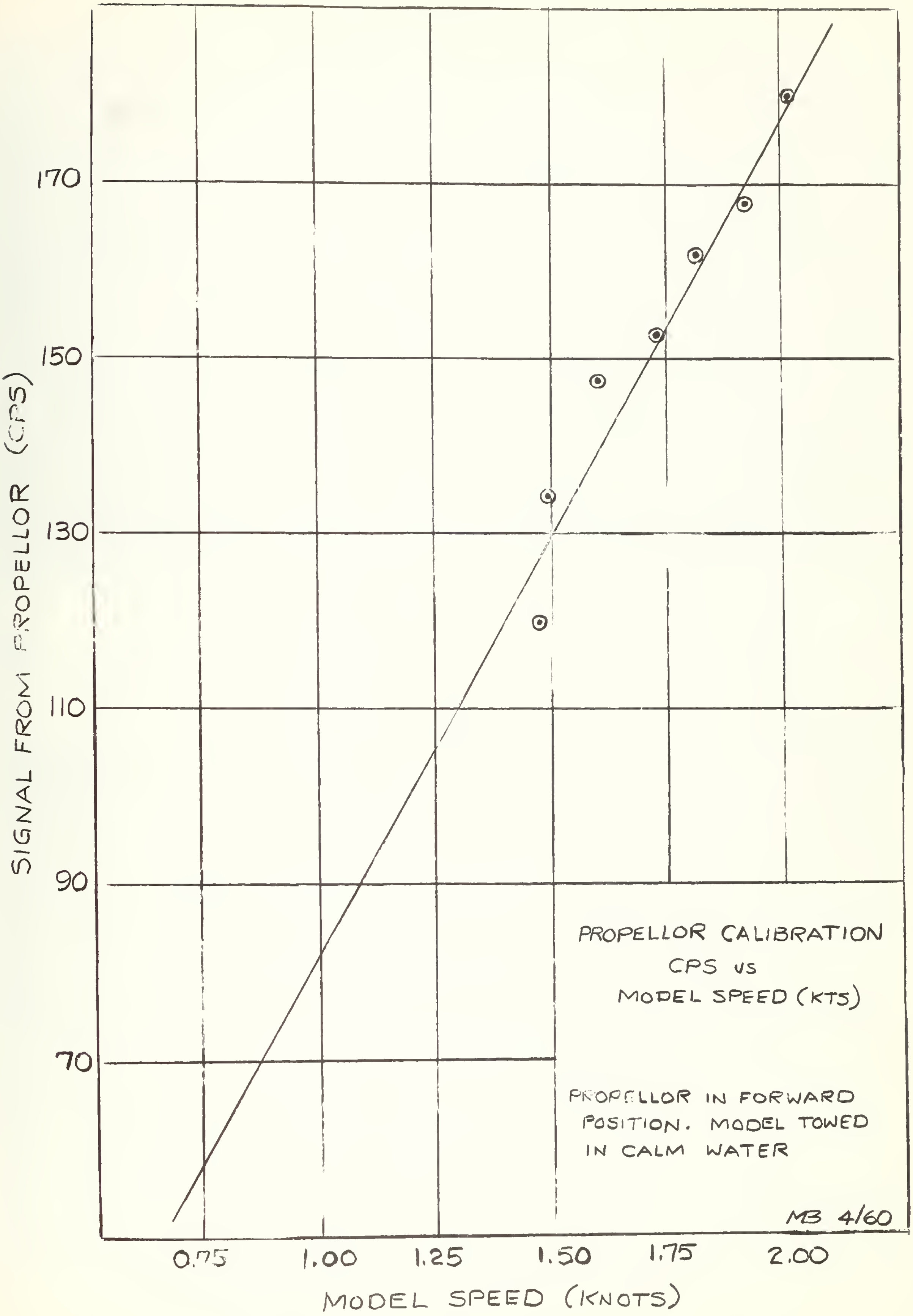


FIGURE 3-I

RUNS 19-20

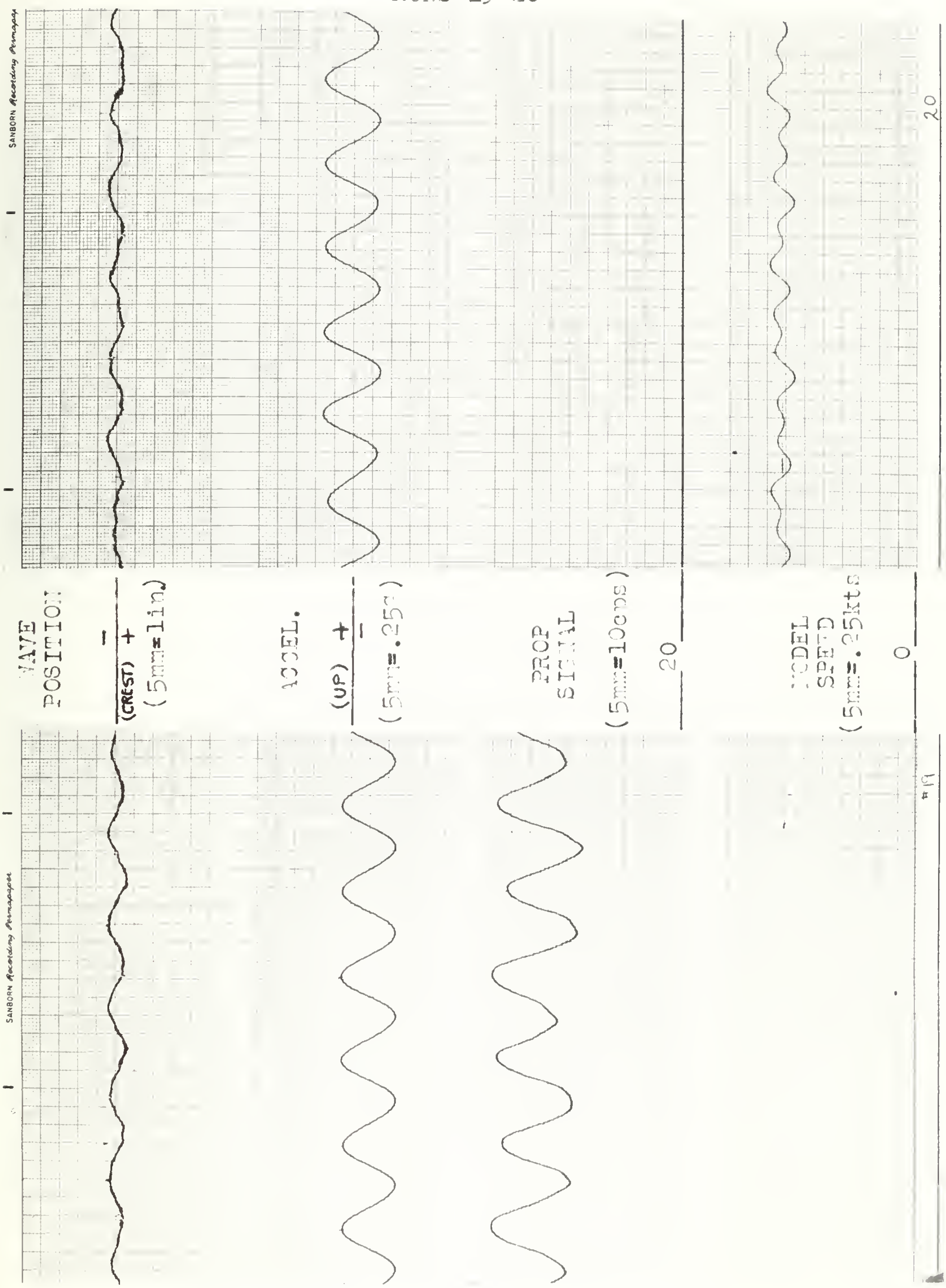


FIGURE B-II

RUNS 21-22

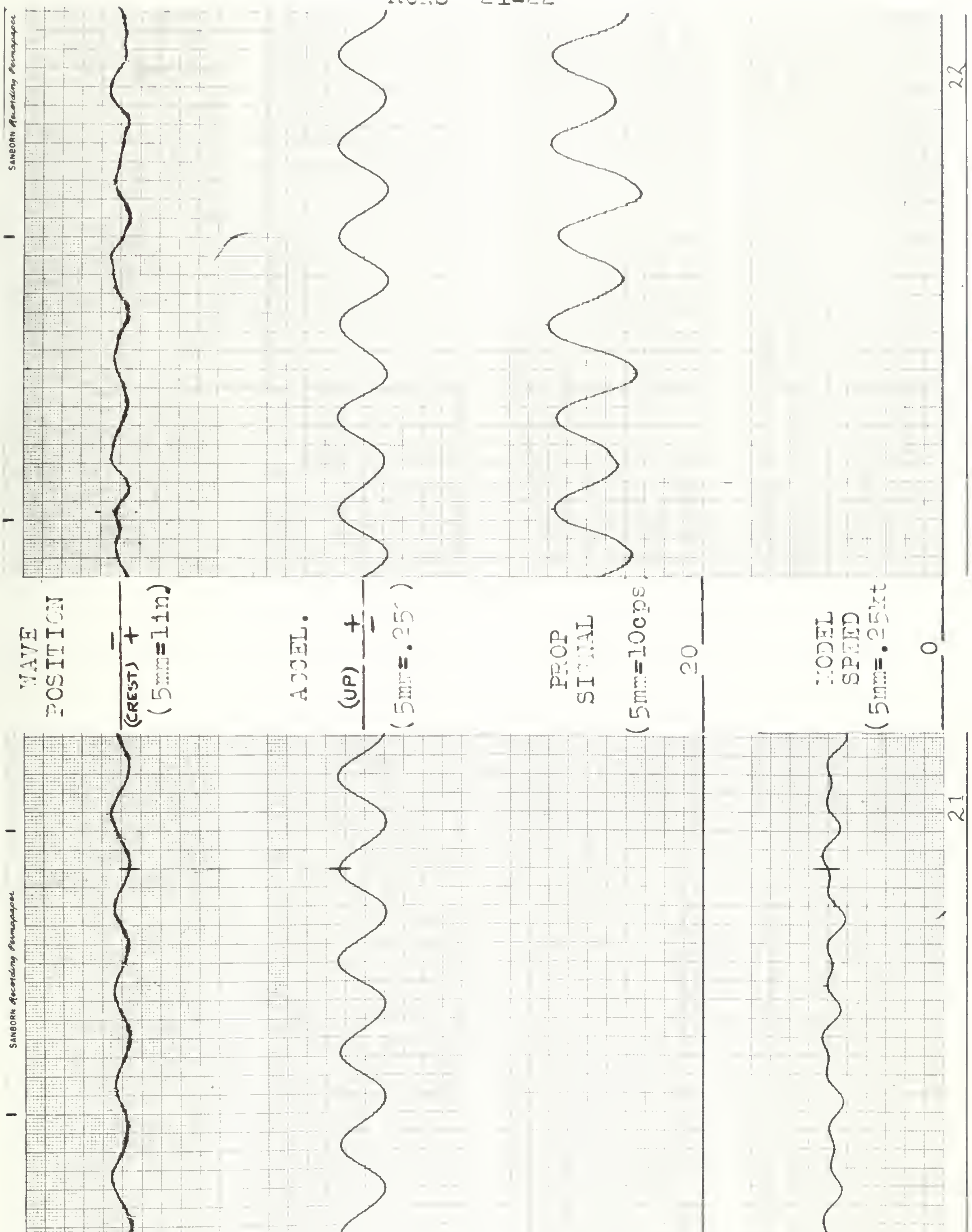


FIGURE B-III

RUNS 31-32

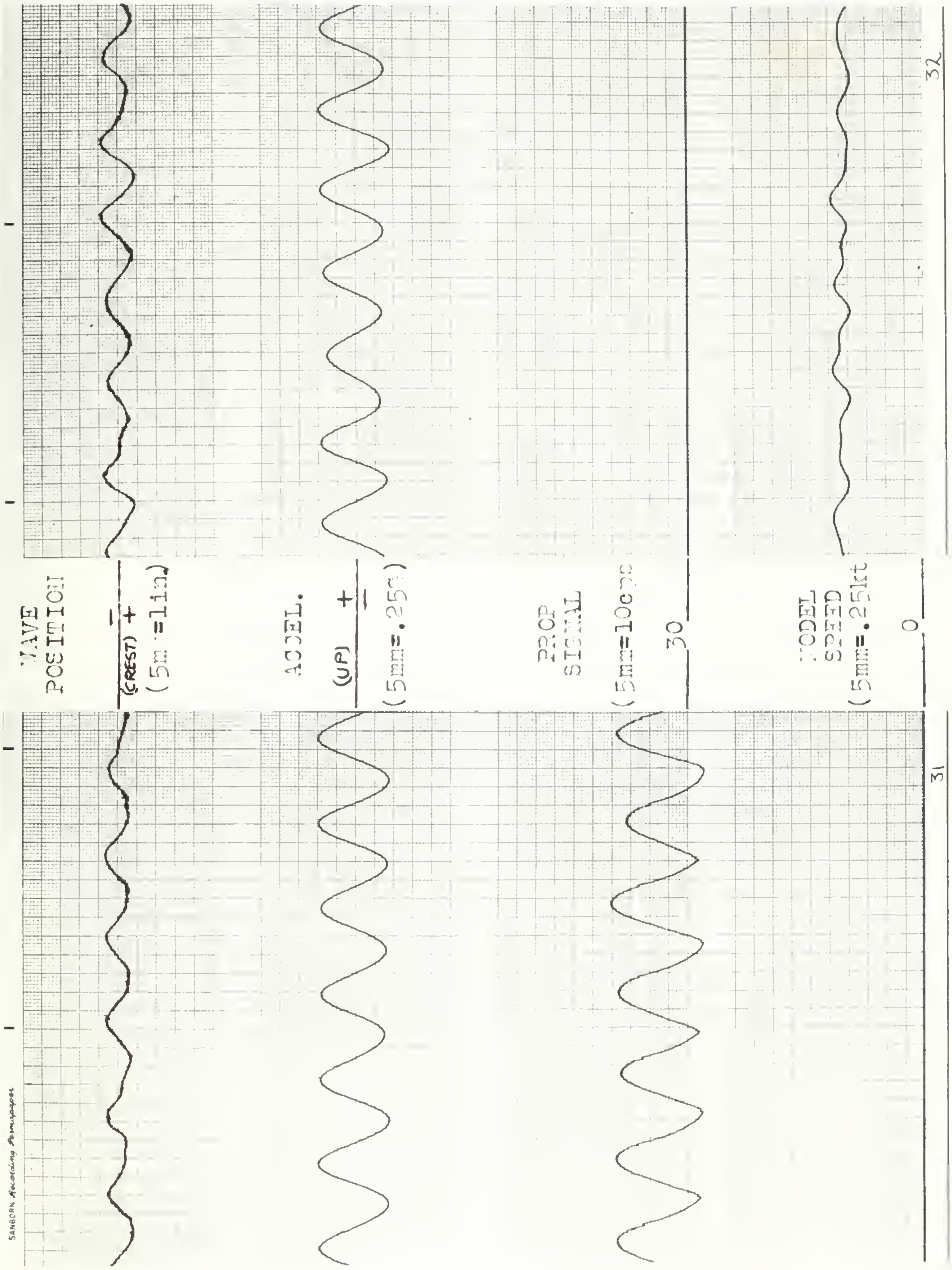


FIGURE B-IV

RUNS 33-34

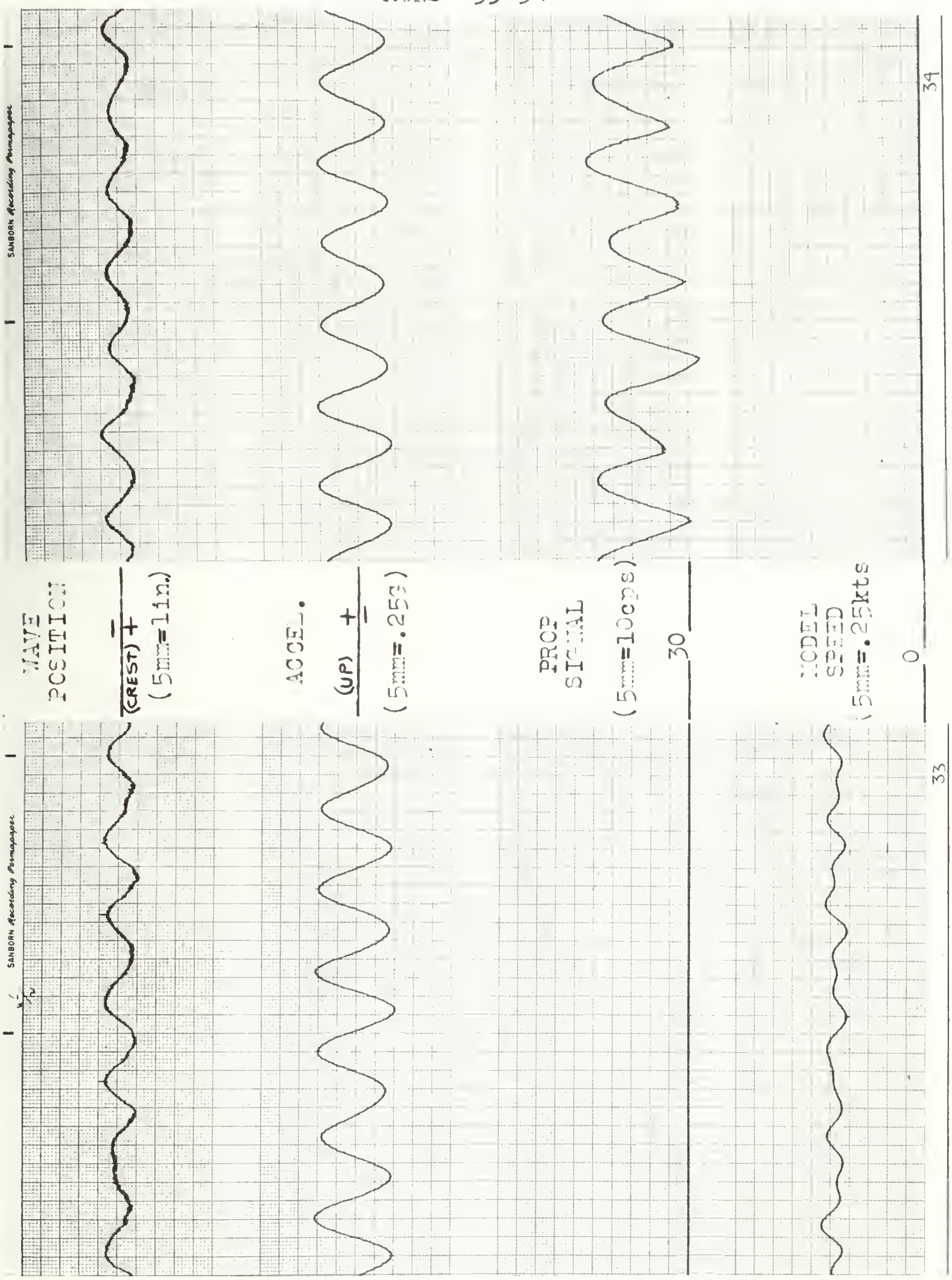


FIGURE 3-V

RUNS 39-40

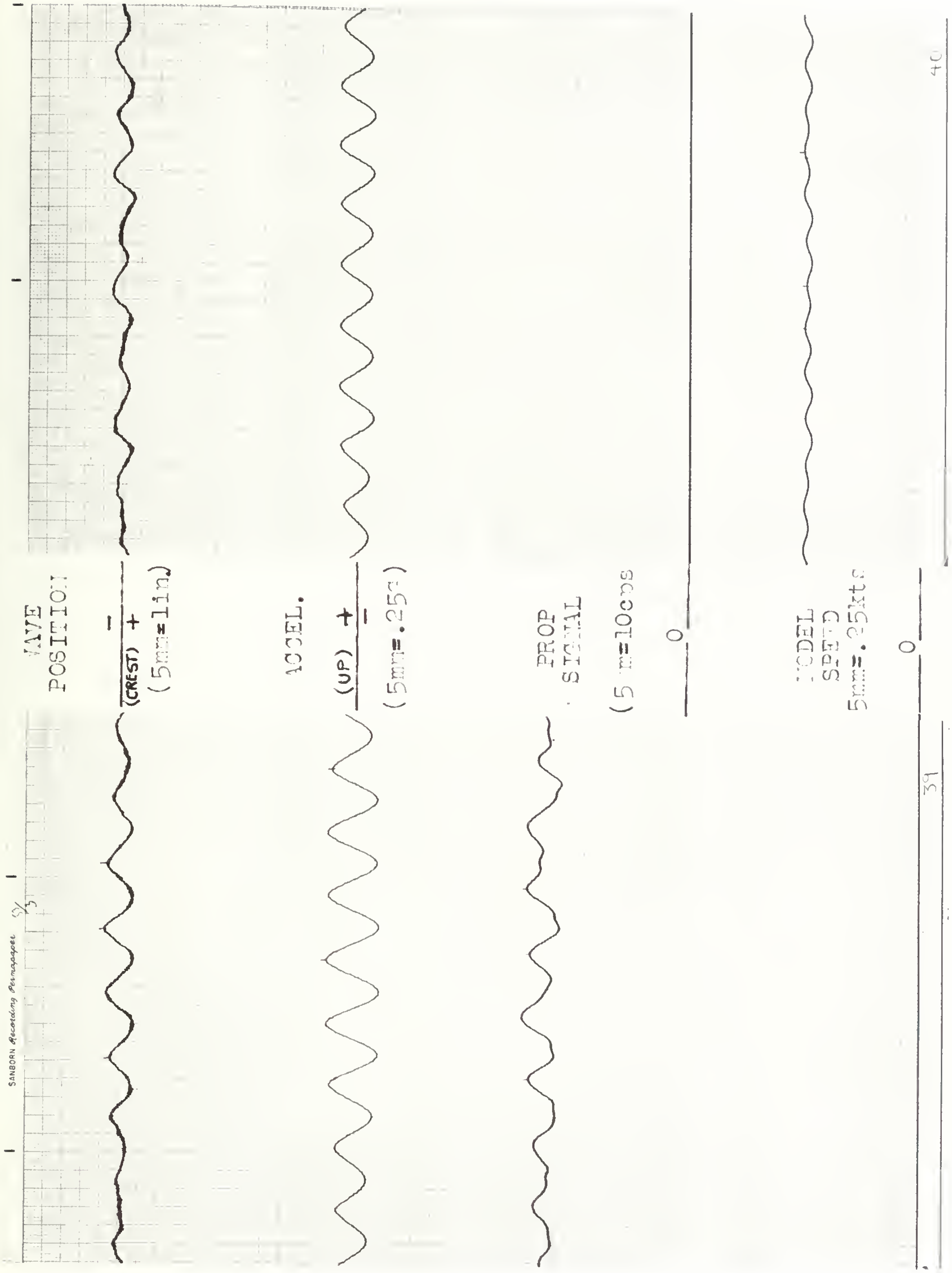


FIGURE B-VI

RUNS 41-42

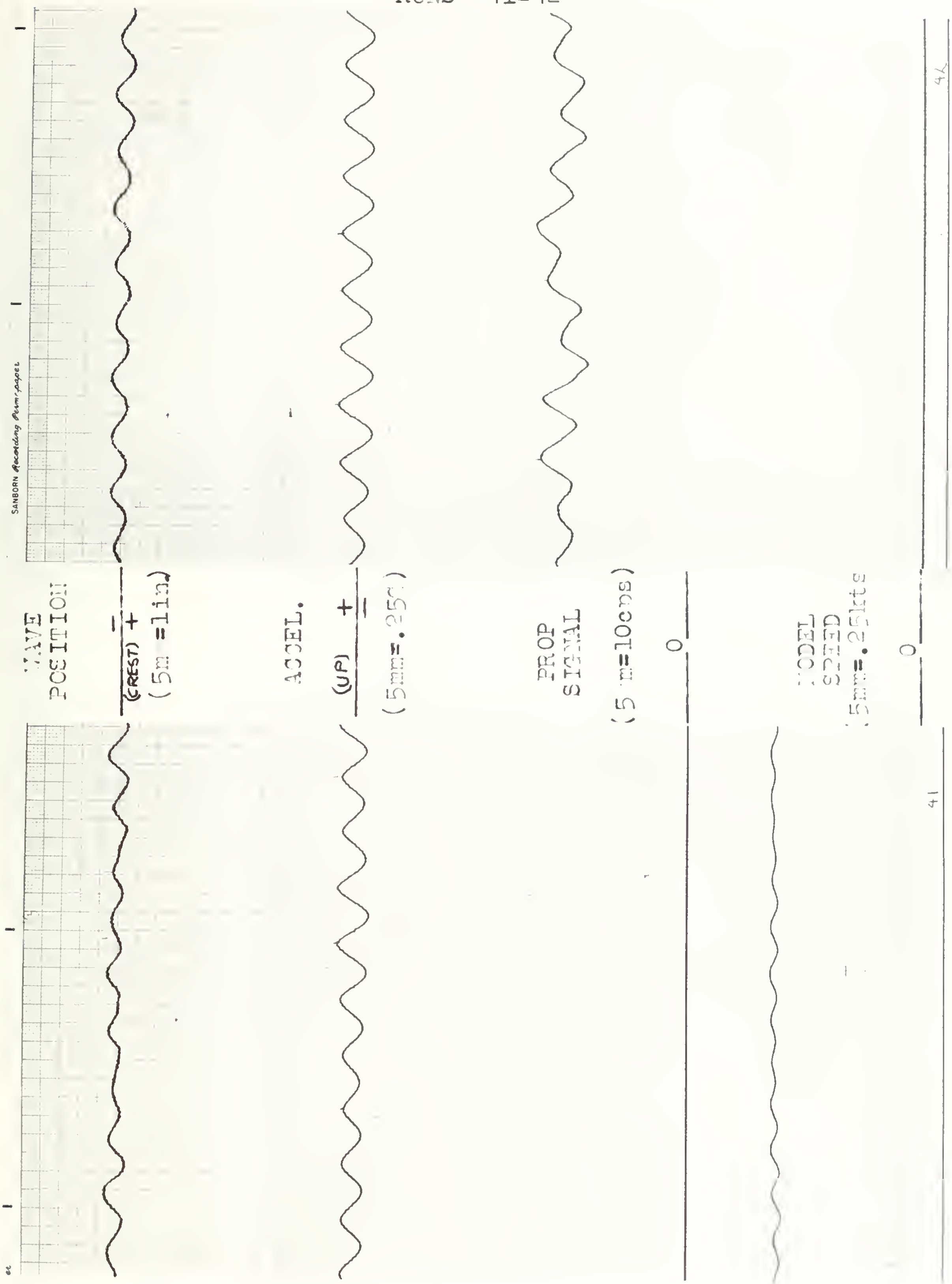


FIGURE B-VII

RUNS 47-49

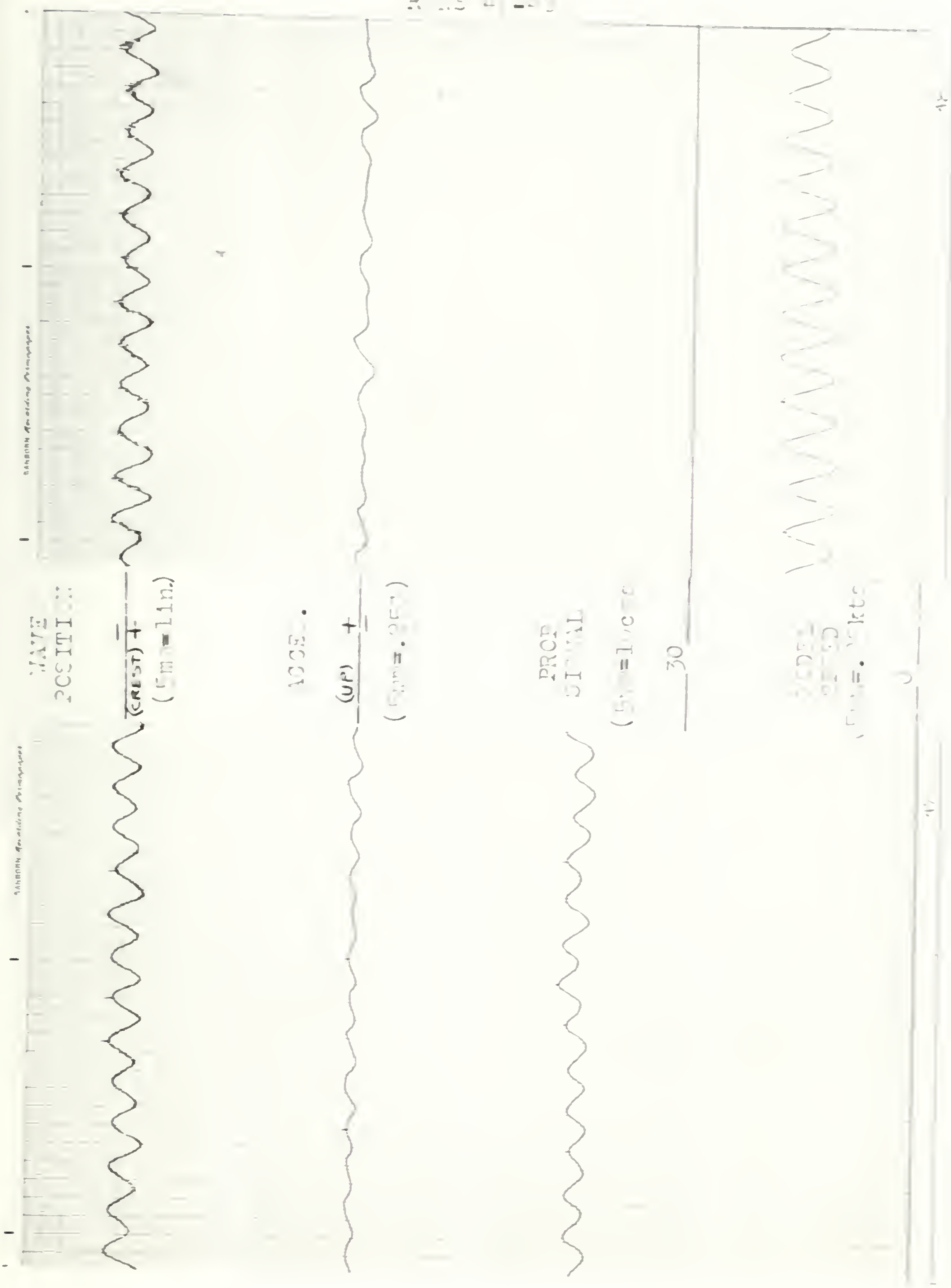


FIGURE B-VIII

RUNS 49-50

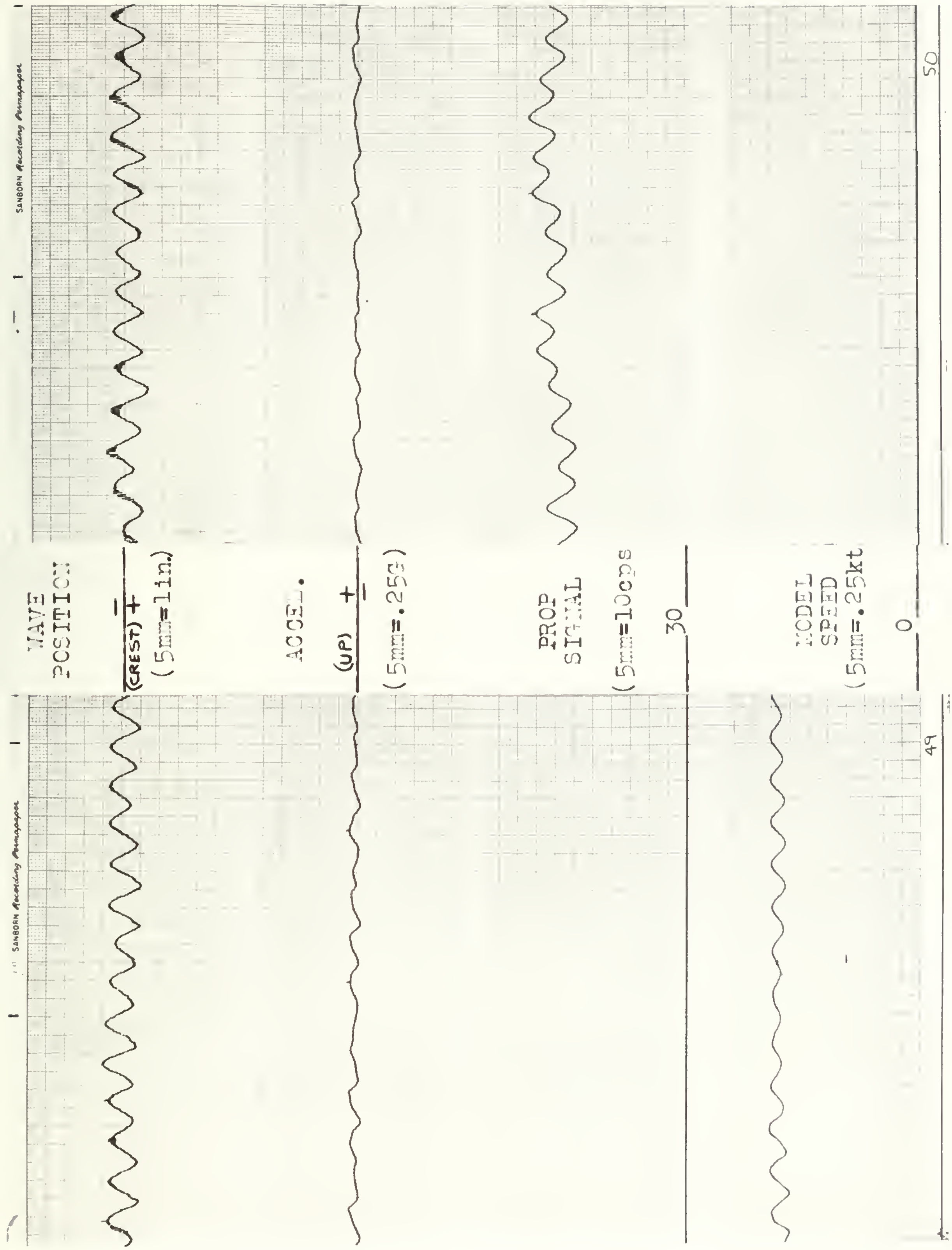


TABLE B-I

Summary of Model Runs

Run	V_b (kts)	CPS_b	w	a (in)	λ (ft)
19-20	2.02	105	0.380	2.5	9.06
21-22	1.78	88	0.396	2.5	9.06
23-24	1.78	88	0.396	1.6	8.85
25-26	2.02	105	0.380	1.6	8.85
27-28	2.02	105	0.380	1.2	7.13
29-30	1.78	88	0.396	1.2	7.13
31-32	1.78	88	0.396	1.4	7.10
33-34	2.02	105	0.380	1.4	7.10
35-36	2.02	105	0.380	2.2	5.05
37-38	1.78	88	0.396	2.2	5.05
39-40	1.78	88	0.396	1.3	5.12
41-42	2.02	105	0.380	1.3	5.12
43-44	2.02	105	0.380	1.4	3.18
45-46	1.78	88	0.396	1.4	3.18
47-48	1.78	88	0.396	1.9	3.15
49-50	2.02	105	0.380	1.9	3.15

V_b Base calm water towing speed.

CPS_b Calm water propeller signal in cycles per second.

w Wake fraction.

a Wave height in inches.

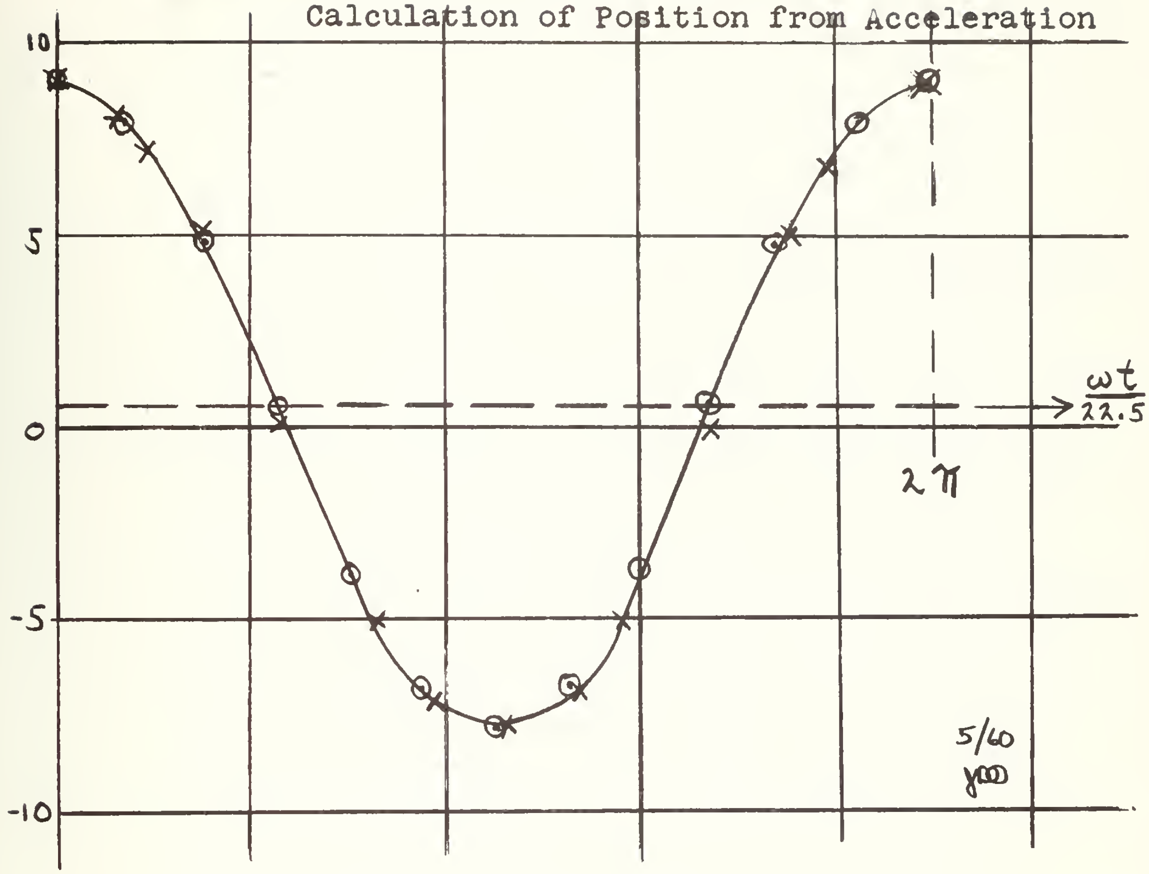
λ Wave length in feet.

APPENDIX C

Calculations

FIGURE C-I

Calculation of Position from Acceleration



⊙—⊙ = 0.5 + cos ωt

×—× = Actual Acceleration Trace (Run #31)

To Compute Displacement:

If Acceleration = A cos ωt

Then, Displacement = $-\frac{A}{\omega^2}$ cos ωt

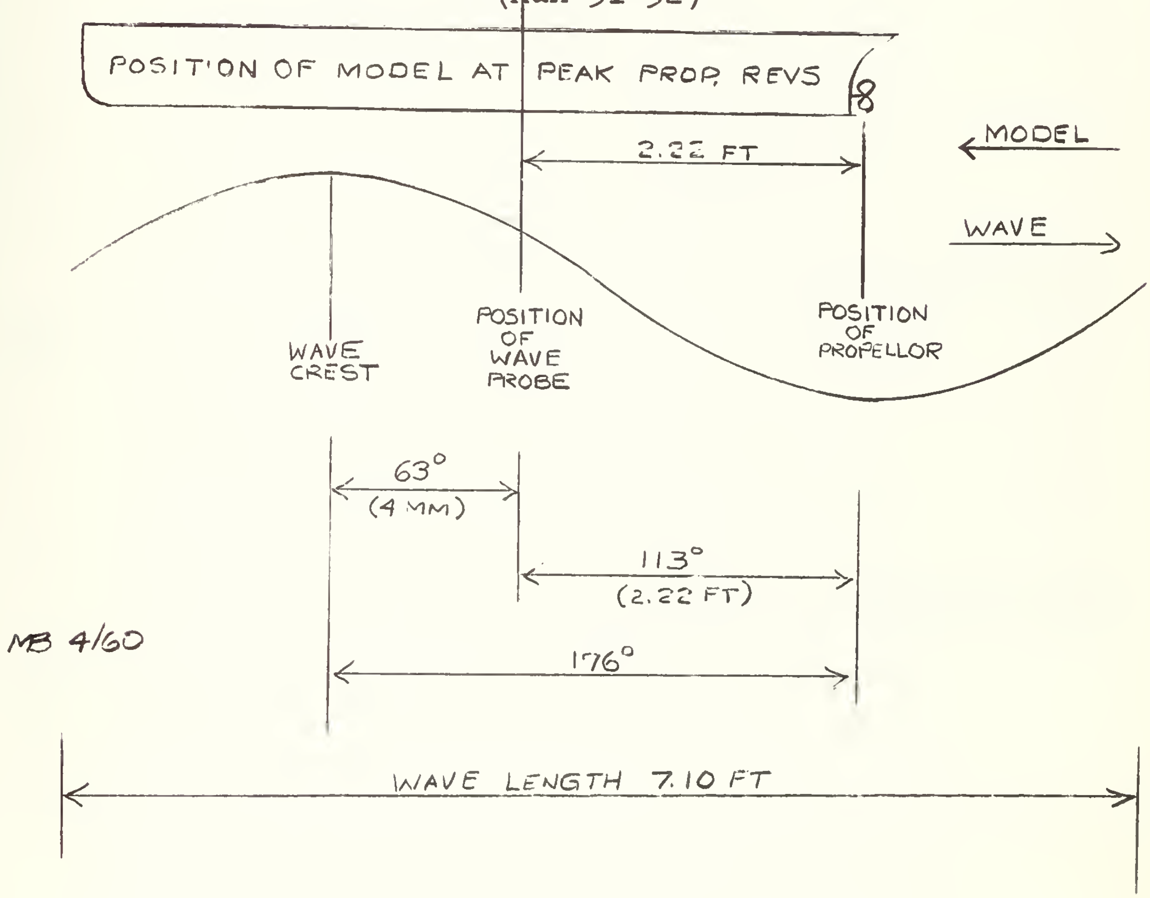
where ω is the frequency of encounter in radians/sec.

C-(2) Calculation of Wave Crest Position Displacement

FIGURE C-II

Correlation of Wave Crest and Propeller Speed Variation

(Run 31-32)



PERIOD OF ENCOUNTER 0.91 SEC (11.4 MM)

WAVE CREST AT PROBE LAGS PEAK PROPELLOR REVS 0.16 SEC (4 MM) = 63°

C-(3) Calculation of Shaft System Frequency Response

1. HUB OF PROPELLER:

$$J_\ell = \frac{m r_\ell^2}{2} \; ; \quad r_\ell = 3/16 \text{ in.} ; \quad m = 1.046 \times 10^{-2} \text{ LBS.}$$

$$\therefore J_\ell = \frac{(1.046 \times 10^{-2})}{2} \times \frac{3^2}{16^2} = \underline{\underline{1.84 \times 10^{-4} \text{ in}^2\text{-LBS}}}$$

2. SPOKES OF PROPELLER:

$$J_s = \frac{N m (\ell_s^2 - r_\ell^2)}{3} \; ; \quad N = 8$$

$m = 1.025 \times 10^{-4}$ LBS

$\ell_s = 1$ in.

$r_\ell = 3/16$ in.

$$\therefore J_s = \frac{8(1.025 \times 10^{-4})(1 - 9/256)}{3}$$

$$= \frac{8(1.025 \times 10^{-4})(.961)}{3} = \underline{\underline{4.10 \times 10^{-4} \text{ in}^2\text{-LBS}}}$$

3. PROPELLER SHAFT:

$$J_{SH} = \frac{m_s r_s^2}{2} \; ; \quad r_s = .0625 \text{ in.}$$

$m_s = 2.14 \times 10^{-2}$ LBS

$$\therefore J_{SH} = \frac{(2.14 \times 10^{-2})(6.25 \times 10^{-2})^2}{2} = \underline{\underline{0.419 \times 10^{-4} \text{ in}^2\text{-LBS}}}$$

4. GEAR:

$$J_g = \frac{m_g r_g^2}{2} \; ; \quad r_g = .3125 \text{ in.}$$

$m_g = 1.598 \times 10^{-2}$ LBS

$$\therefore J_g = \frac{(1.598 \times 10^{-2})(3.125 \times 10^{-1})^2}{2} = \underline{\underline{7.8 \times 10^{-4} \text{ in}^2\text{-LBS}}}$$

5. VANES:

$$J_V = \frac{Nm}{12}\left[4(\ell_s+a)^2+4t^2-(4\ell_s^2+t^2)\right]$$

$N = 8$; $m = 8.0\times10^{-4}$ LBS.

$\ell_s = 1$; $a = 0.4$ in ; $t = 0.02$ in.

WHERE: a AND t ARE THE LENGTH AND THICKNESS OF THE VANE.

$$\therefore J_V = \frac{8(8.0\times10^{-4})}{12}\left[\cancel{4\ell_s^2}+8\ell_s a+4a^2+4t^2-\cancel{4\ell_s^2}+t^2\right]$$

$$= \frac{64\times10^{-4}}{12}\left[8(1)(.4)+4(.16)+3(4\times10^{-4})\right]$$

$$= 5.34\times10^{-4}\left[3.2+.64+.0012\right]$$

$$= 5.34\times10^{-4}\left[3.8412\right]$$

$$J_V = \underline{\underline{20.5\times10^{-4}\ \text{in}^2\text{-LBS.}}}$$

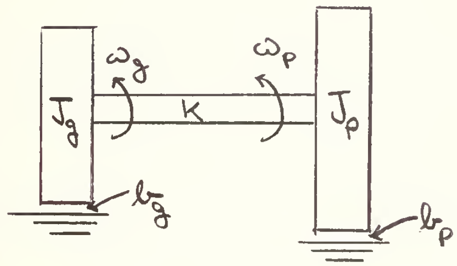
6. TOTAL INERTIA:

$$J_T = J_a + J_s + J_{SH} + J_g + J_V$$

$$= (1.84+4.10+0.419+7.8+20.5)\times10^{-4}$$

$$\underline{\underline{J_T = 34.659\times10^{-4} = .034659\ \text{in}^2\text{-LBS}}}$$

7. IF SHAFT IS CONSIDERED TO BE A SIMPLE TWO MASS SYSTEM:



J_p & J_g = MASS MOMENT OF PROP. & GEAR (IN-LBS-SEC^2)

K = SPRING CONSTANT OF SHAFT (IN-LBS/RAD.)

b_p & b_g = DAMPING AT PROP. AND GEAR (IN-LBS.-SEC).

ω_p & ω_g = ANGULAR VELOCITY (RAD/SEC).

USING LAPLACE TRANSFORM NOTATION.

$$J_p s\omega_p + b_p\omega_p + \frac{K}{S}(\omega_p - \omega_g) = T \qquad (C-1)$$

$$J_g s\omega_g + b_g\omega_g + \frac{K}{S}(\omega_g - \omega_p) = 0 \qquad (C-2)$$

WHERE: T = EXCITATION TORQUE AT PROP.

COMBINING (C-1) AND (C-2) GIVES:

$$\frac{\omega_g}{T} = \frac{K}{S^3 J_g J_p + S^2(J_g b_p + J_p b_g) + S\left[K(J_g + J_p) + b_g b_p\right] + K(b_g + b_p)} \qquad (C-3)$$

IF IT IS ASSUMED THAT THE FRICTION TERMS (b_p & b_g) ARE VERY SMALL COMPARED TO THE OTHER PARAMETERS THE FOLLOWING APPROXIMATE RELATIONSHIP IS OBTAINED:

$$\frac{\omega_g}{T} = \frac{1}{(J_p + J_g)\left[S^2 \frac{J_g J_p}{K(J_g + J_p)} + 1\right]} \qquad (C-4)$$

THEREFORE:

$$\omega_m^2 = \frac{K(J_g + J_p)}{J_g J_p} \qquad (C-5)$$

WHERE: ω_m = NATURAL FREQUENCY OF THE SYSTEM.

HOWEVER:

$$K = \frac{G J_s}{L}$$

G = MODULUS OF ELASTICITY (LBS/IN^2)

J_s = POLAR MOMENT OF SHAFT (IN^4)

L = LENGTH OF SHAFT.

SO: $$K = \frac{(12\times10^6)(2.47\times10^{-5})}{6.5}$$

$$K = 45.6 \text{ IN-LB/RAD}$$

SUBSTITUTING NUMBERS IN (C-5).

$K = 45.6$; $J_g = 2.02\times10^{-6}$; $J_p = 6.74\times10^{-6}$

$$\omega_m^2 = \frac{45.6\,(8.76\times10^{-6})}{13.6\times10^{-12}}$$

$$\omega_m^2 = 29.4\times10^6$$

$$\underline{\underline{\omega_m = 5.4\times10^3}}$$

8. The above analysis neglects any effect from the entrained water of the propeller. It can be seen, from equation (C-5) that if I_p is increased, in the limit, $\omega_n^2 = K/I_g$ which is still on the order of 10^6 so the natural frequency will still be in the kilocycle range.

APPENDIX D

Investigation of Errors in Results

The results obtained during sixteen model runs indicated there were one or more basic errors present in the measurement system. Careful study of the Sanborn Recordings and the results derived from these recordings led to the conclusion that the following errors were possible:

(1) Large surges in model speed.

(2) Phase inversion in one or more of the systems.

(3) Phase lags (time constants) in one or more of the systems.

The possibility that large surges in model speed affected the magnitude of the wake variation was easily checked. Two simple tests, covered in the Discussion of Results, showed that variation from the mean model speed was negligible, and had little or no effect on the accuracy of the wake measurements.

The other two possible errors required more detailed analysis before their effects could be determined. This procedure will be covered in detail in the discussion to follow.

D-(1). Phase Inversion

It was easily determined that there was no chance of a complete 180° phase reversal occurring in the propeller revolution system. This left only the wave position and acceleration systems to be investigated. In both, the measurement is a variation from a steady state value, and this deviation could be, and was, both positive and negative in polarity.

Acceleration measurements were made with the Recorder zeroed at the normal earth's gravitational force. The accelerometer used was of the strain-gage-variable resistance type. It was noted in the course of the experimental work that the polarity of the output signal was dependent on the physical orientation of the accelerometer. The dependence is easily understood by referring to Figure D-I. In this figure, the accelerometer is represented by a single wire, whose resistance is proportional to length. If this sensitive wire is fixed to the accelerometer case at one end and attached to a small weight free to move in one direction at the other, it becomes immediately obvious that a sudden acceleration of the case, for example, in an upward direction, will increase the resistance of the element if the accelerometer is oriented in one manner, and decrease the resistance if oriented in an opposite manner.

This dependence was important because the accelerometer was installed upside down during many of the model runs to improve the stability characteristics of the model.

As a check that the polarity of the acceleration signal and the motion of the model were in agreement, visual observations in conjunction with the manually operated timing marker on the Recorder were made. In relatively long waves the motion of the model was very pronounced, and it was a simple matter to note the time when the stern was either in a maximum up or down position. These observations definitely established the proper phase calibration of the acceleration circuit. They also showed that there was little or no time delay between the motion and reaction of the Recorder stylus.

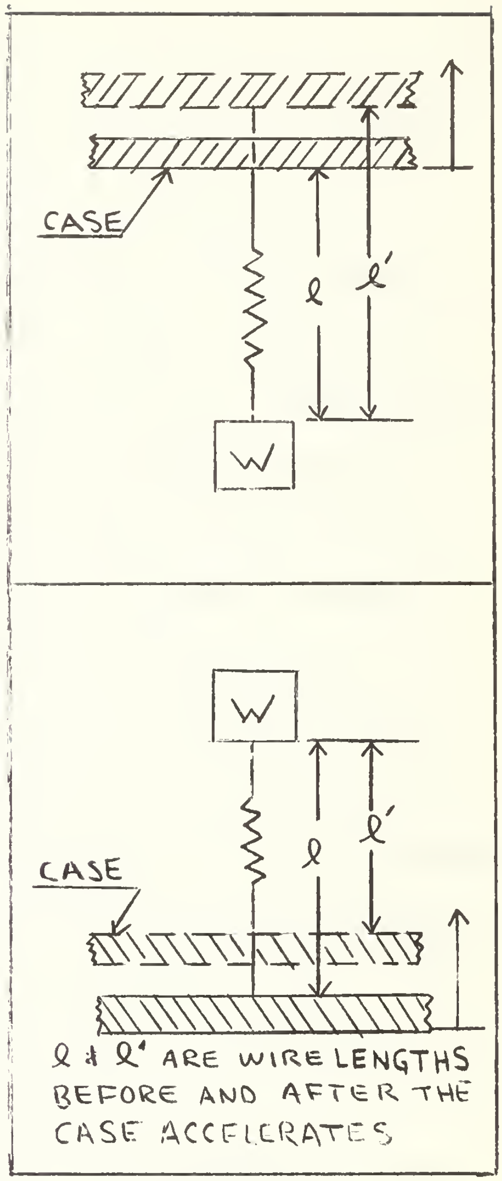


FIGURE D-I

The wave probe was zeroed at the calm water waterline. An initial check on the polarity of the signal was made when the probe was mounted on the side of the tank to record the wave

lengths and heights. By use of the timing marker, and visually noting when the wave crest was at the probe, it was definitely established that increasing the immersion of the probe (at a wave crest) caused a downward deflection of the stylus.

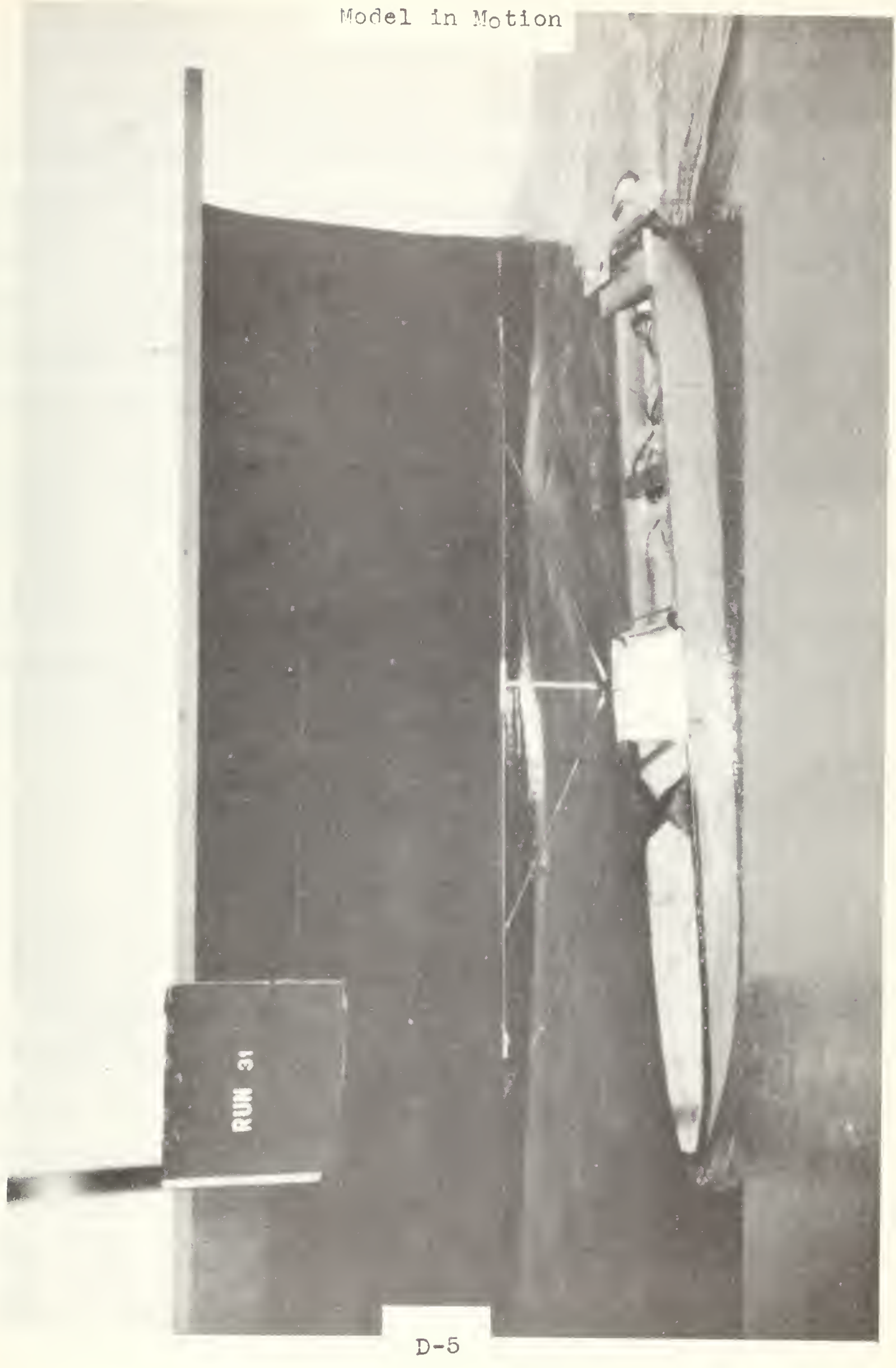
To show that these two recording circuits were in agreement, the wave probe was relocated at the stern, and runs in wave lengths greater than model length were made. Under these conditions, the immersion of the probe was dependent on the motion of the stern rather than on the position of the wave. During the course of these runs, a Polaroid camera was used to photograph the model in motion. By a series of these pictures, an example of which is reproduced as Figure D-II, it was confirmed that for long wave lengths the stern of the model was at or near its maximum down position when the trough of the wave was at the propeller location. This result checks the relationships shown for the long wave lengths in Figures XI-XVI, and is also in agreement with the studies conducted by Korvin-Kroukovski and Jacobs (8).

Therefore it can be definitely stated that there are no significant phase shifts or magnitude errors in either the measurement of acceleration or wave height.

There remains the possibility that the position of the wave crest determined from the wave probe recording is in error as a result of model motion. During the actual run, the probe was located approximately amidships, where the

Figure D-II

Model in Motion



only motion experienced was heave. It is for this reason that it is strongly recommended that future studies be conducted in a facility with a carriage above the tank, and the wave probe be suspended from this carriage to eliminate any vertical motion. As an alternative, an accelerometer mounted amidships could be used to measure heave. This would require additional recording channels, which are not presently available at the M.I.T. Towing Tank.

D-(2). Frequency Characteristics

It was a simple matter to check the wave and acceleration systems for polarity and phase shift. Any frequency dependence in these circuits must result from line capacitance in the long cable. This effect is discounted for two reasons:

(1) The frequency at which the signals varied was extremely low (0.5-5 CPS).

(2) All four signals were subject to the same cable run, and thus would be subject to the same attentuation.

The wake measurement circuit is not so simple to analyze. Complete phase reversal was easily eliminated as a possibility. In the combined mechanical-electrical system, a frequency cut-off could occur in the region of operation.

In the original analysis the mechanical frequency response (effect of inertia, shaft length, etc.) was estimated and found to be satisfactory (Appendix C). Electrically there was

little chance of time lag between pickup and Recorder. As a check on the electrical characteristics, a step change of frequency was introduced into the counter, and the Recorder response noted. While this was a very crude way of measuring frequency response, it did show that the electrical time constant was at least an order of magnitude less than that which would be necessary to account for the phase lags in the results.

The greatest unknown in the overall frequency response was the relationship between the wake velocity and the torque applied to the rotating propeller. Also unknown was the resistance torque of the bearings and of the rotating propeller in water. The next step was to investigate these factors more thoroughly.

To show if there was a phase shift in the overall system, a series of model runs were made with the propeller located in the calibrating position (at the bow). This placed the propeller in what was essentially an area of undisturbed flow. The model was run in regular waves through a wide range of wave lengths (16.6-2.8 feet), and by a combination of visual observations with the timing marker, and the wave probe located amidships, the relationship between propeller RPM variation and wave crest position was determined. If the original assumption that variation in wake velocity was caused by orbital velocity was correct, then the propeller should speed up at the wave crests and slow down in the troughs.

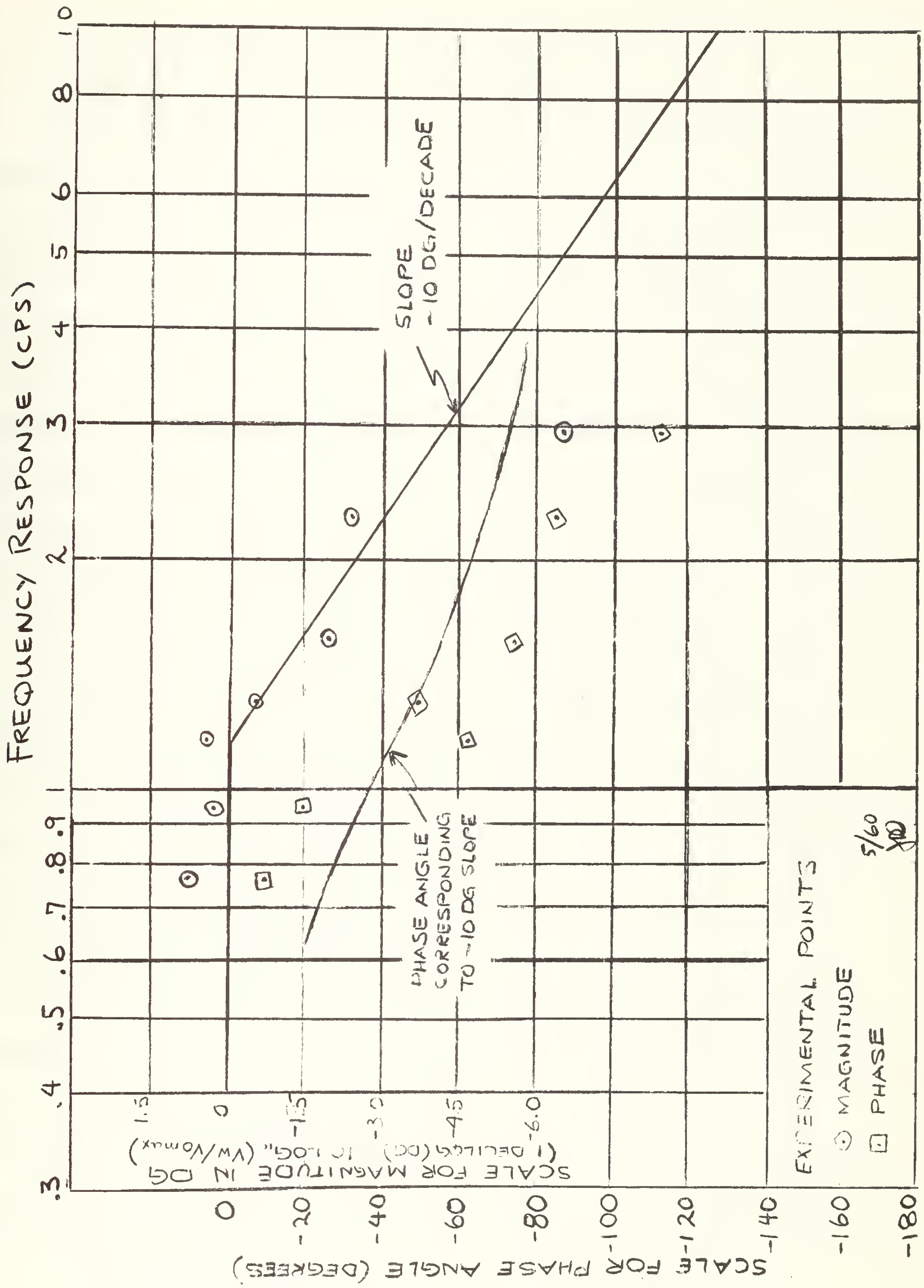
It was first noted that for the longest wave length the two recordings were very nearly in phase. At the shortest wave length, there was a large phase difference. To determine if there was any pattern to these shifts, a frequency response plot was constructed (Figure D-III). For this plot it was assumed that the excitation was the maximum orbital velocity, and that zero phase shift would be when maximum RPM coincided with the position of the wave crest at the propeller. This assumption is not exactly correct, since the orbital velocity is a maximum only at the surface. The entire propeller certainly does operate in this region, and in these runs care was taken to insure the propeller was always completely immersed. This method of normalization did serve to provide a common basis for comparison.

Figure D-III, though it covers only a small region of the frequency spectrum, definitely indicated that some sort of frequency cut-off existed in the region of one cycle per second frequency of encounter. Extending this range was not possible. Maximum model speed limited the high frequencies which could be tested. Signal strength from the pickup was the limit on low frequencies.

Once it had been ascertained that some kind of frequency limitation existed, it was necessary to determine the likely cause. This involved considering the system as a whole, with input as wake velocity and output as the voltage from the pickup (proportional to shaft speed). This required that the

FIGURE D-III

Frequency Response of Propeller System



inertia calculations be repeated to include variables unknown at the time of the original calculations.

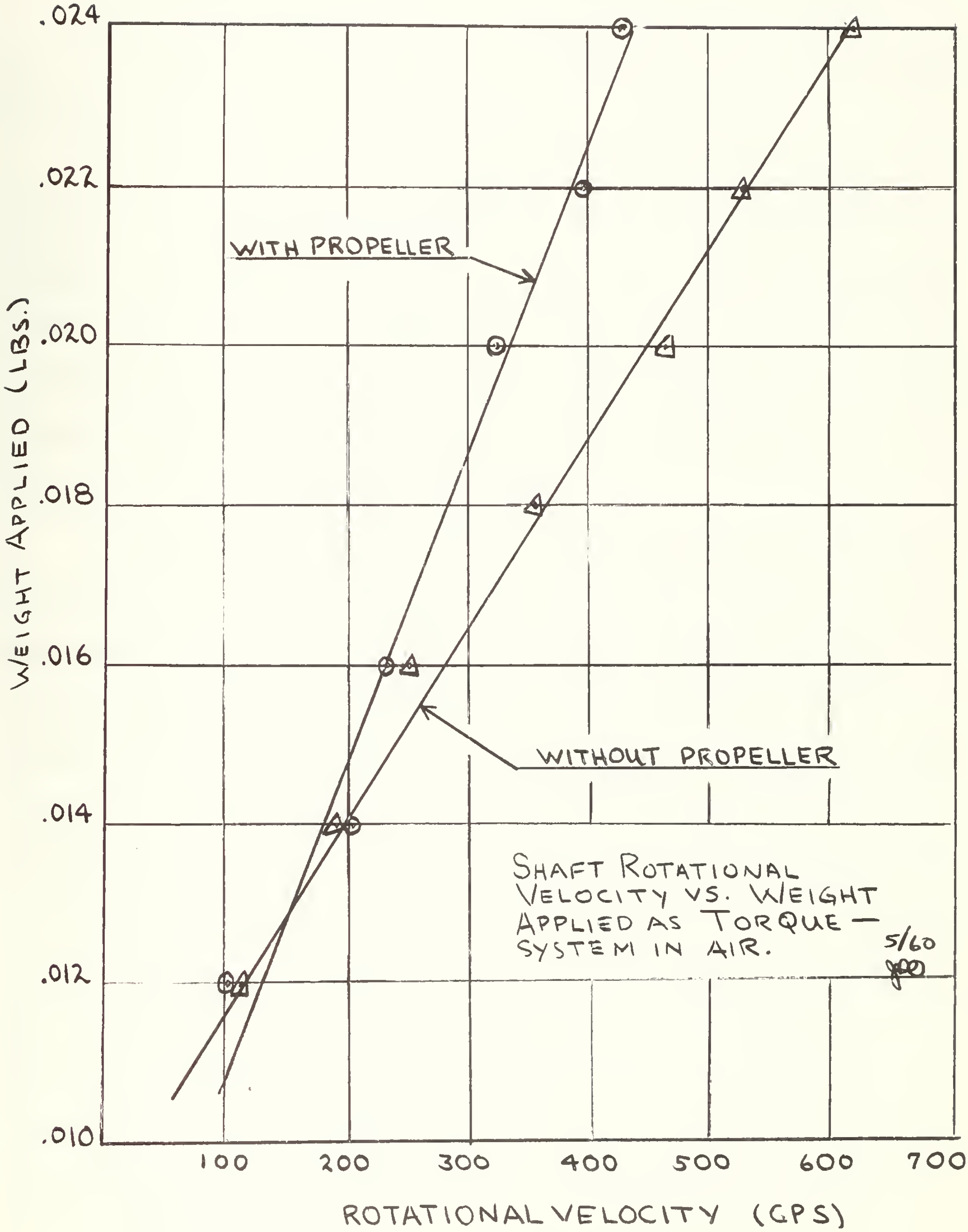
Tests were made with the model out of water to determine bearing friction and shaft assembly inertia. Small weights suspended from a length of light thread wound around the shaft were dropped, and the resulting shaft speed recorded. When the shaft reached a steady speed, it was assumed that the torque applied by the freely falling weight was exactly balanced by the friction torque in the bearings. These tests were made with the propeller both on and off the shaft. The difference in steady state shaft speed in the two cases was attributed to air resistance of the rotating vanes.

The results of these tests are shown in Figure D-IV. Some propeller air resistance is definitely indicated, particularly at the higher rotational speeds. At lower speeds the two curves tend to merge, as would be expected. From the data obtained from the runs without the propeller mounted, the bearing friction torque was computed to be proportional to speed.

These same results also provide a check on the original estimate of shaft inertia. It may be assumed that the applied torque of the falling weight is resisted by two torques. One is bearing friction (proportional to rotational speed), and the other is the torque required to overcome the inertia of system when accelerated. Initially when the speed

FIGURE D-IV

Propeller Resistance in Air



is zero, the slope of the RPM vs. time recording is proportional to the inertia of the system.

Inertia calculations and results are summarized in Table D-II. They show reasonable agreement with the original estimates in Appendix C.

The same procedure was used with the propeller immersed in water. Again the steady state RPM plotted against the torque applied (Figure D-V) shows the relationship between speed and resistance torque. The slope of the RPM vs. time recording is proportional to the total inertia of the system, this time including the entrained water. The inertia calculations are shown in Table D-I.

It should be noted that the values computed are based on data obtained with the model motionless. The friction and inertia are both subject to correction when the propeller has a non-zero speed of advance. In the case of inertia, it would be expected that the effect of added mass of the entrained water would be independent of speed of advance (14). Figure D-VI shows that it is not independent of rotational speed. This may be accounted for in part by the crude method of performing these tests. The weights were dropped in water, and while the weight in water applied could be accurately computed the resistance of the water to varying sized objects could not. It seems reasonable to assume that the torque computed for the heavier weights,

TABLE D-I

Calculation of Propeller Inertia in Water

W	Slope	dw/dt	T	J
0.430	75/2	117.8	0.0271	0.0893
0.344	112/3	117.2	0.0216	0.0775
0.258	110/3	115.0	0.0162	0.0544
0.102	85/3	89.0	0.0108	0.0473
0.086	75/4	59.0	0.0054	0.0353

W Effective weight in water.

Slope Initial slope of propeller speed trace (mm/sec)

dw/dt Angular acceleration (radians/sec/sec)

T Torque applied to propeller shaft (in-lbs)

J Total inertia of assembly (in^2-lbs)

TABLE D-II

Calculation of Propeller Inertia in Air

W	Slope	dw/dt	T	J
0.012	75/5	47.0	0.000755	0.0062
0.014	150/11	42.8	0.000882	0.00795
0.022	130/6	72.0	0.001384	0.00742
0.024	200/8.5	74.0	0.00151	0.00788

W Weight (lbs)

J Inertia of shaft assembly in air (in^2-lbs)

Slope, dw/dt, and T same as in Table D-1. Radius at which W applied was 0.0625 inch for both.

FIGURE D-V

Propeller Resistance in Water

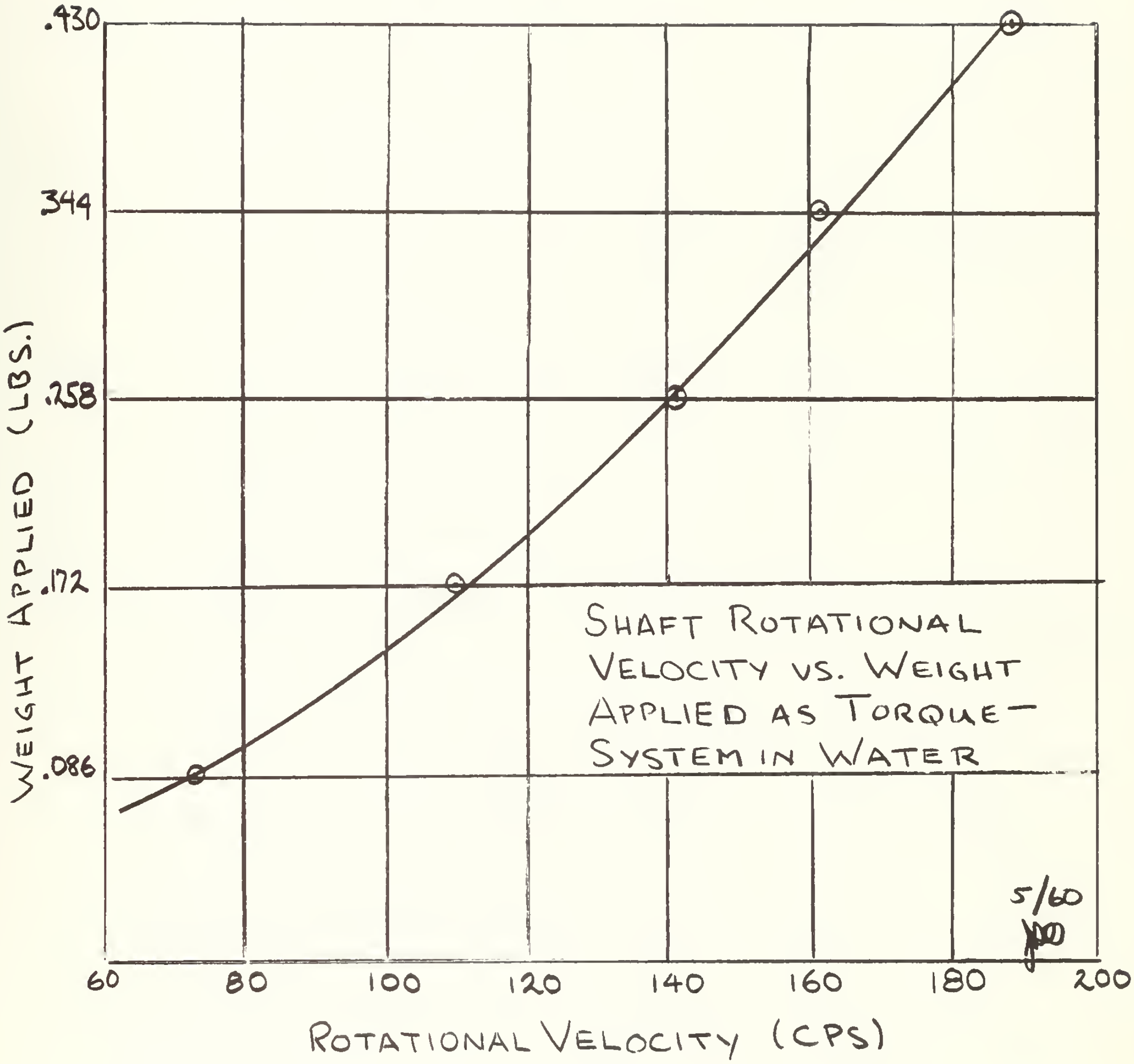
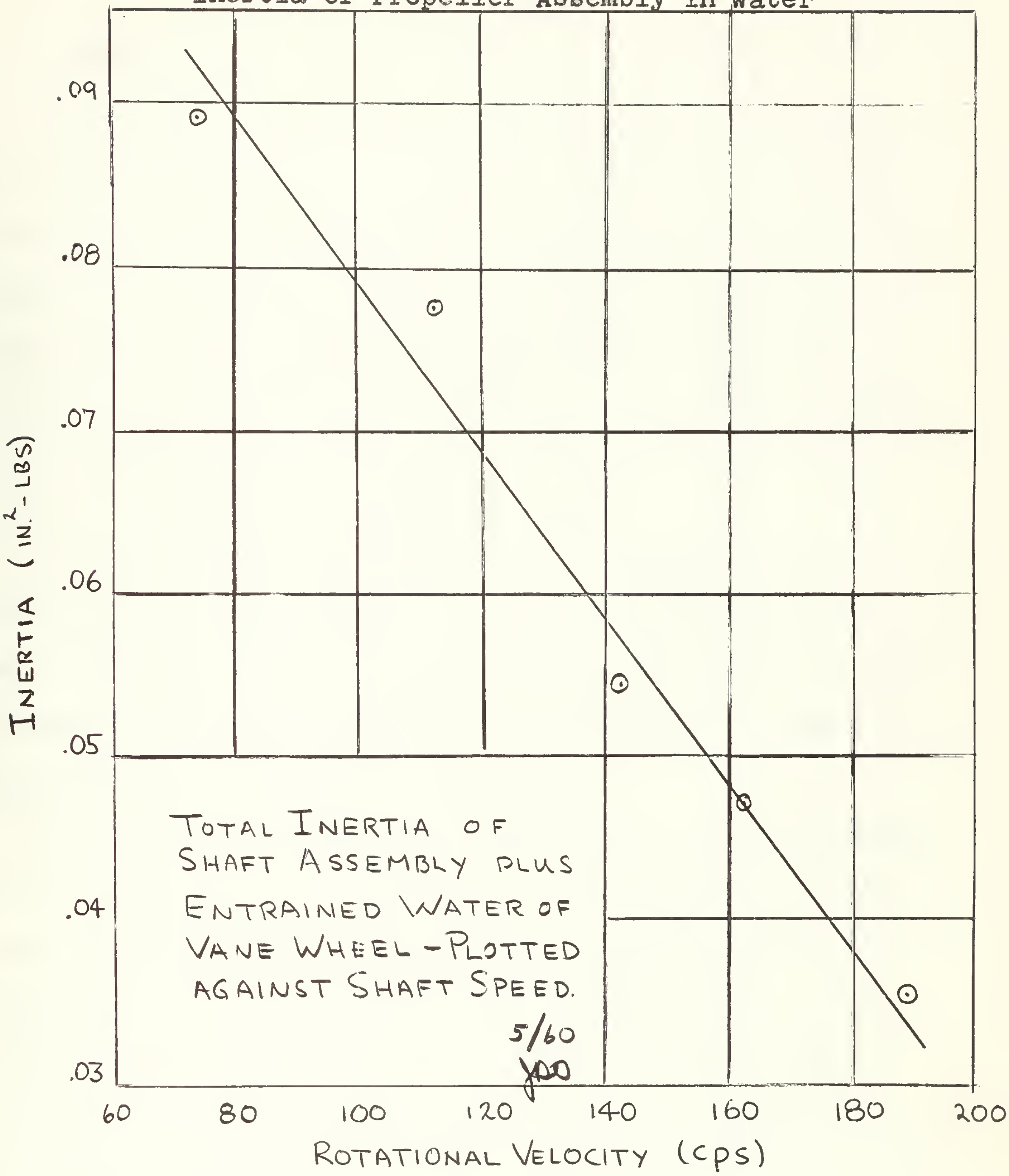


FIGURE D-VI

Inertia of Propeller Assembly in Water



based only on the weight of the object in water, was in fact less than this amount by the magnitude of the form resistance of the weight.

While these tests (using falling weights to determine resistance coefficients and inertia were not of sufficient accuracy to completely determine the exact magnitude of the unknown variables in the system, they did give an order of magnitude figure. The values determined verified the original assumption that the inertia of the shaft assembly itself would be unimportant when compared to the inertia of the system with its entrained water.

With the figures for resistance and torque, it was planned to return to the points plotted in Figure D-III and determine if the same characteristics could be obtained by analytical means. The analytical analysis is shown in Appendix D-(3).

An attempt was also made to measure the torque response of the propeller to a change in speed of advance, but in the limited time available, the results were largely inconclusive. The method used was to run the model in calm water. After a steady speed was reached a small weight, attached to a thread wound around the shaft was picked up. It was hoped that the new steady state RPM would give some measure of the speed-torque relationship.

Equipment installed at the tank limited the height to which

the weight could be raised to about twelve inches. This small distance made it impossible to insure that a new steady state speed had been reached.

The shaft arrangement required that the tests be conducted with the thread wound around the only exposed portion of the shaft, between the outboard bearing and the propeller. This meant the thread was exposed to the water velocity, and great difficulty was experienced in making the thread wind evenly and smoothly. Use of an inboard section of the shaft would have been more satisfactory, but this would have required dismantling and rebuilding the shaft assembly, which time did not permit.

Even without this last bit of experimental data, it is possible to approximate the frequency response of the shaft system. To simplify the calculations, all equations have been linearized. The following section summarizes these calculations, the results of which are included in Figure D-III.

D-(3). Frequency Response Calculations

To analyze the frequency response characteristics of the propeller system, it is necessary to carefully consider all the possible variables. Simplifications are in order only when their use will not materially affect the results.

The calculations to follow will be presented in Laplace Transform notation. Desired is the overall transfer function

of the propeller system, which may be expressed as:

$$H(S) = \frac{\omega(S)}{V_w(S)} = \frac{\omega(S)}{T(S)} \times \frac{T(S)}{V_w(S)} \qquad \text{(D-1)}$$

With approximations and linearization where necessary, this transfer function will be expressed in the form:

$$H(S) = \frac{A}{\tau S+1} \qquad \text{(D-2)}$$

This leads to a time solution as follows:

(1) Input: Step of magnitude V.

Output: $\omega(t) = VA(1 - \varepsilon^{-t/\tau}) \qquad$ (D-3)

(2) Input: $V_w = V \sin \omega t$

Output: $$\omega(t) = \frac{VA}{\sqrt{\tau^2\omega^2 + 1}} \sin(\omega t + \phi) \qquad \text{(D-4)}$$

where $\phi = \tan^{-1} \omega\tau$

One further convention will be used in these calculations. All variables will be treated as a sum of a steady-state value, denoted by the subscript 0, and an incremental change from this steady-state, denoted by the prefix Δ. The final solution will express the relationship only between incremental variables. At any given operating point the steady-state terms balance, and can be cancelled out of the equation.

Thus the simple relationship $F = Ma$ may be expressed as:

$$F_o + \Delta F = m(a_o + \Delta a) \qquad \text{(D-5)}$$

For an incremental relationship, the steady-state terms are dropped, leaving:

$$\Delta F = m(\Delta a) \qquad \text{(D-6)}$$

All the calculations in this section are based on the velocity vector diagram shown in Figure D-VIII, and on the block diagram in Figure D-VIII.

The analysis is based on the assumption that two independent velocity changes determine the propeller angular speed. For an incremental change of wake velocity, initially there is no change in propeller speed, which means that the incremental component of the tangential velocity is zero. The additional torque that results from this change provides the acceleration to the propeller. As the propeller changes speed, there is no further change in wake velocity. Thus the torque change resulting from an incremental change in tangential velocity can be determined as occurring with the incremental wake velocity equal to zero.

The following basic equations and relationships are used:

$$\frac{\Delta \omega(s)}{\Delta T(s)} = \frac{1}{J\ s+b} \qquad \text{(D-7)}$$

FIGURE D-VII

Propeller Velocity Vector Diagram

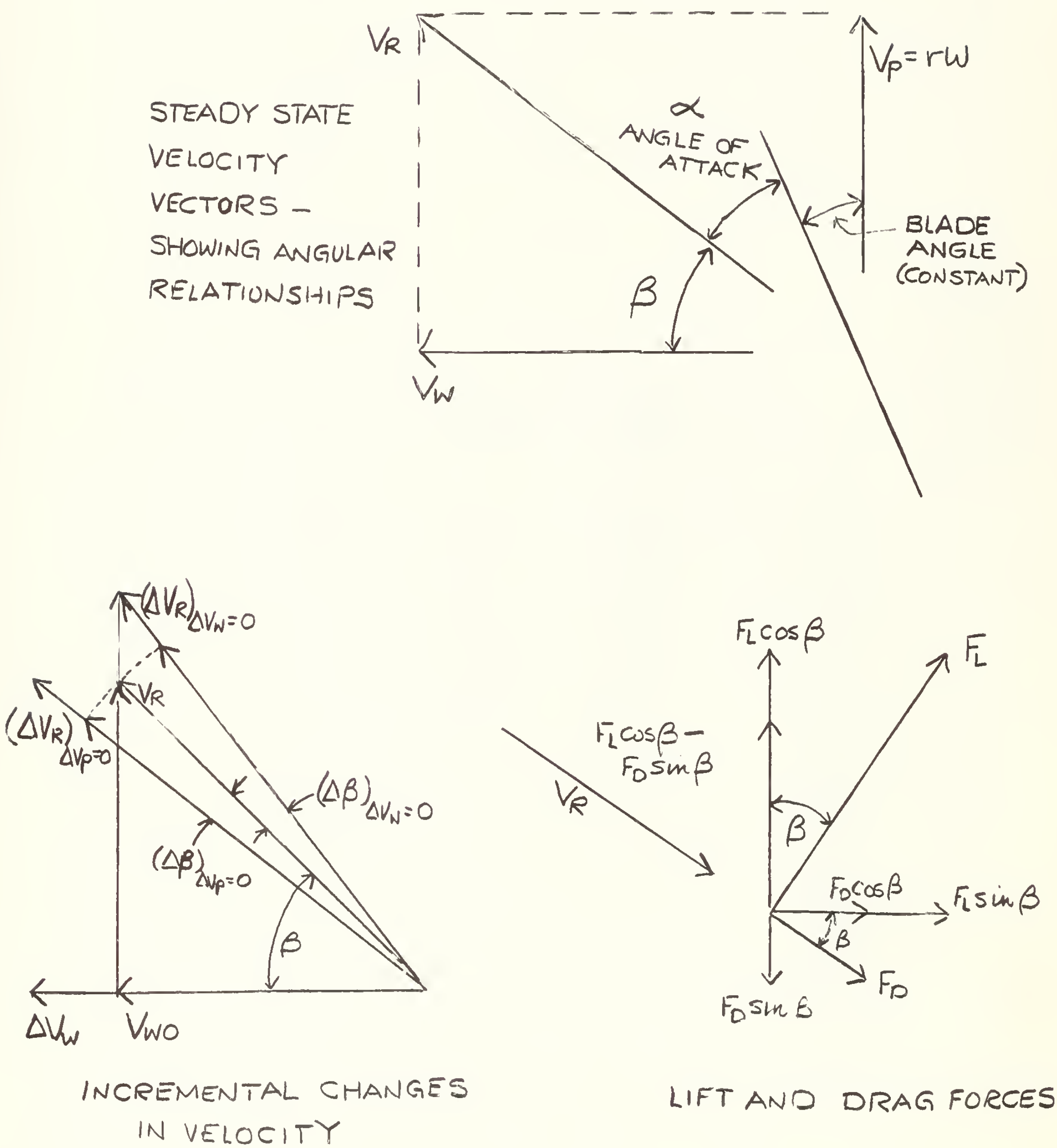


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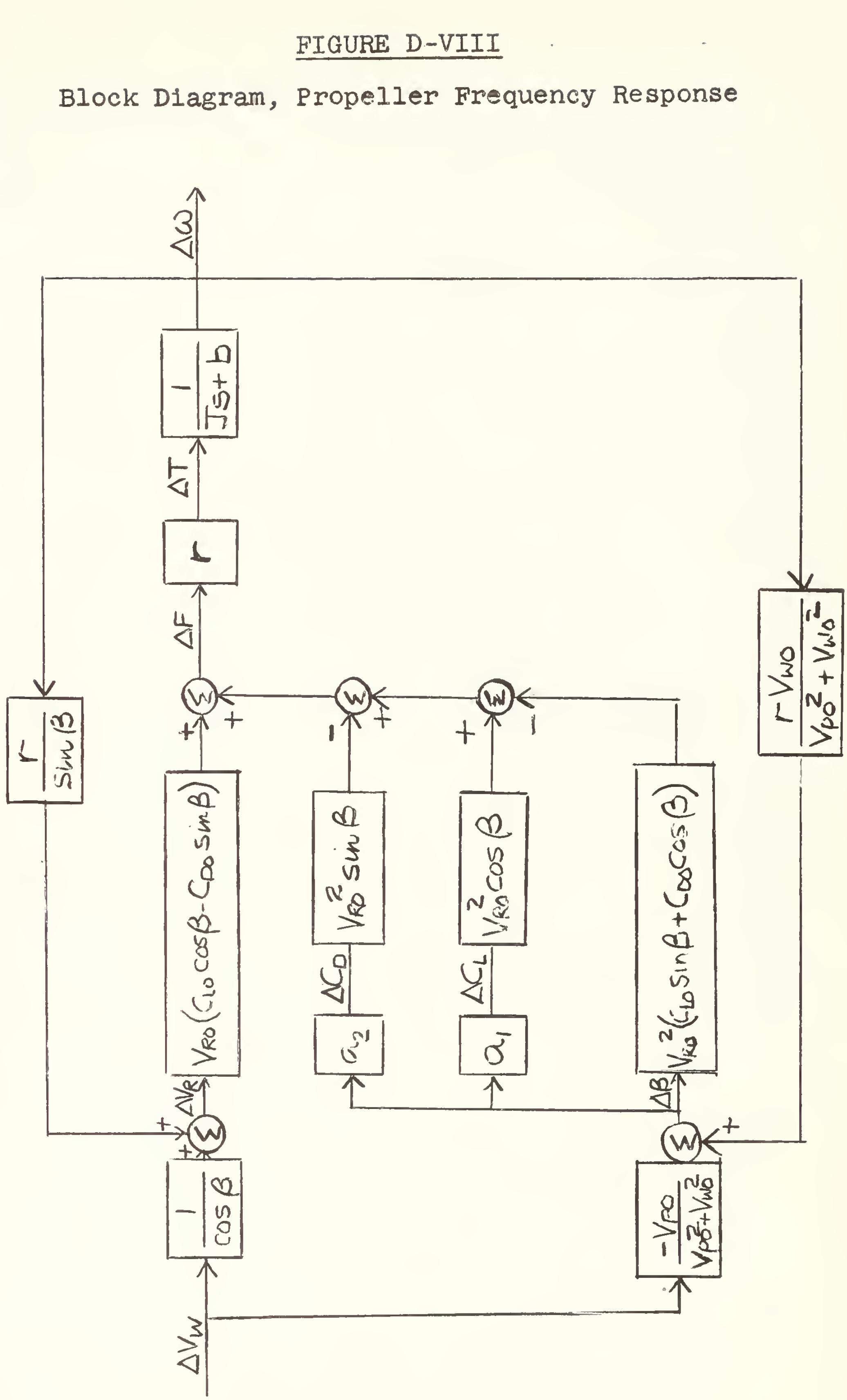
FIGURE D-VIII

Block Diagram, Propeller Frequency Response



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$$F_L = C_L V_R^2 \tag{D-8}$$

$$F_D = C_D V_R^2 \tag{D-9}$$

$$F_1 = F_{L1} + F_{D1} = F_L \cos \beta - F_D \sin \beta \tag{D-10}$$

$$T = F_1 r \tag{D-11}$$

$$\beta = \tan^{-1} \frac{V_p}{V_w} \tag{D-12}$$

where the symbols used may be defined:

T = Hydrodynamic torque (in-lbs)

ω = Propeller angular speed (rad/sec)

J = Mass moment of inertia of propeller system (in-lbs-sec^2)

b = Bearing friction (in-lbs-sec)

F_1 = Airfoil lift force (lbs)

F_D = Airfoil drag force (lbs)

C_L = Lift coefficient, including all constant terms in lift equation

C_D = Drag coefficient

r = Mean radius of propeller (in)

β = Angle between wake velocity (V_w) and tangential velocity of propeller (V_p)

α = Angle of attack on airfoil

The change in the angle β which occurs as a result of changes in the two velocity vectors may be computed:

$$\Delta\beta = \frac{-V_p}{V_w^2 + V_p^2} \Delta V_w + \frac{V_w}{V_w^2 + V_p^2} \Delta V_p \qquad \text{(D-13)}$$

Equation (D-10) can be put in the form of equation (D-5):

$$F_{10} + \Delta F_1 = (C_L + \Delta C_L)(V_{RO} + \Delta V_R)^2 \cos(\beta + \Delta\beta)$$

$$- (C_D + \Delta C_D)(V_{RO} + \Delta V_R)^2 \sin(\beta + \Delta\beta) \qquad \text{(D-14)}$$

Expanding equation (D-14) and dropping the steady-state and higher order incremental terms, an equation for the incremental change in tangential force is left:

$$\Delta F_1 = (V_{RO}^2 \cos\beta)\Delta C_L - (V_{RO}^2 \sin\beta)\Delta C_D \qquad \text{(D-15)}$$

$$+ \left[(C_{LO} \cos\beta - C_{DO} \sin\beta) 2V_{RO}\right] \Delta V_R$$

$$- \left[(C_{LO} \sin\beta + C_{DO} \cos\beta) V_{RO}^2\right] \Delta\beta$$

Figure D-VII also shows that:

$$\Delta V_R \approx \left[\frac{\Delta V_w}{\cos\beta}\right]_{\Delta V_p = 0} \qquad \text{(D-16)}$$

$$\Delta V_R \approx \left[\frac{\Delta V_p}{\sin\beta}\right]_{\Delta V_w = 0}$$

We may also assume that for a small range of angle of attack:

$$\Delta C_L = a_1 \Delta\beta$$

$$\Delta C_D = a_2 \Delta\beta \qquad \text{(D-17)}$$

where a_1 and a_2 are constants.

The block diagram may be reduced to give an expression of the form of equation (D-2). Of interest in this analysis is the time constant, which is expressed as:

$$\tau = \frac{J}{b + r^2\left[\frac{V_{RO}}{\sin\beta}(C_{LO}\sin\beta - C_{DO}\sin\beta) - V_{wo}(a_2 + C_{LO})\sin\beta + V_{wo}(a_1 - C_{DO})\cos\beta\right]} \qquad \text{(D-18)}$$

This expression shows that the time constant in air, which is the simple expression J/b, is modified by the hydrodynamic terms in the denominator. This hydrodynamic term should be as large as possible for a short time constant. While all the terms are interrelated, a small steady-state angle β is an obvious requirement. As might be suspected, the overall problem is not so simple. A small angle β means that the tangential speed is small with respect to wake velocity, which means low angular speed of the propeller. The difficulty encountered with low propeller speed and the accompanying low

signal strength has been discussed in other sections.

D-(4). Conclusions

It would be possible to compute a numeric solution to equation (D-18). This would have little value, and certainly in this case, little accuracy. The data which is particularly lacking is the values of lift and drag coefficients. Comparison of the values of wake and tangential velocity actually encountered in the experimental runs shows that the angle of attack was in the region of 20°-30°. This means that the vanes were not operating in the normal airfoil data region.

This analytical analysis does indicate that the relationship between wake velocity and propeller speed is extremely important in the frequency response of the system. The question of the angle at which the propeller vanes should be set is definitely far more important than was originally assumed when assembling the apparatus.

APPENDIX E

Investigation of Other Causes of Wake Variation

It is not possible to explain the total phase lag that occurs in the results by the apparent frequency response characteristics discussed in Appendix D. This led to an investigation of other possible causes of the variation in wake velocity.

The other variable measured during the regular test runs, and not considered in any of the calculations to this point, was acceleration (position) of the stern. Although the propeller itself is insensitive to velocities in a plane parallel to the plane of rotation, there were indications that this vertical motion was in some way affecting the recorded RPM measurement. Two observations during the course of the regular runs led to this conclusion:

(1) The results (Figures XI-XVIII) show that the peak wake velocity closely followed the motion of the stern. In many ways there was better correlation between stern motion and wake variation than between wave crest location and wake.

(2) In calm water, a small pitching or heaving motion caused very definite variations in the wake record-

ing. This motion was irregular, and could not be controlled, because it was caused by the weight of the cable support pole.

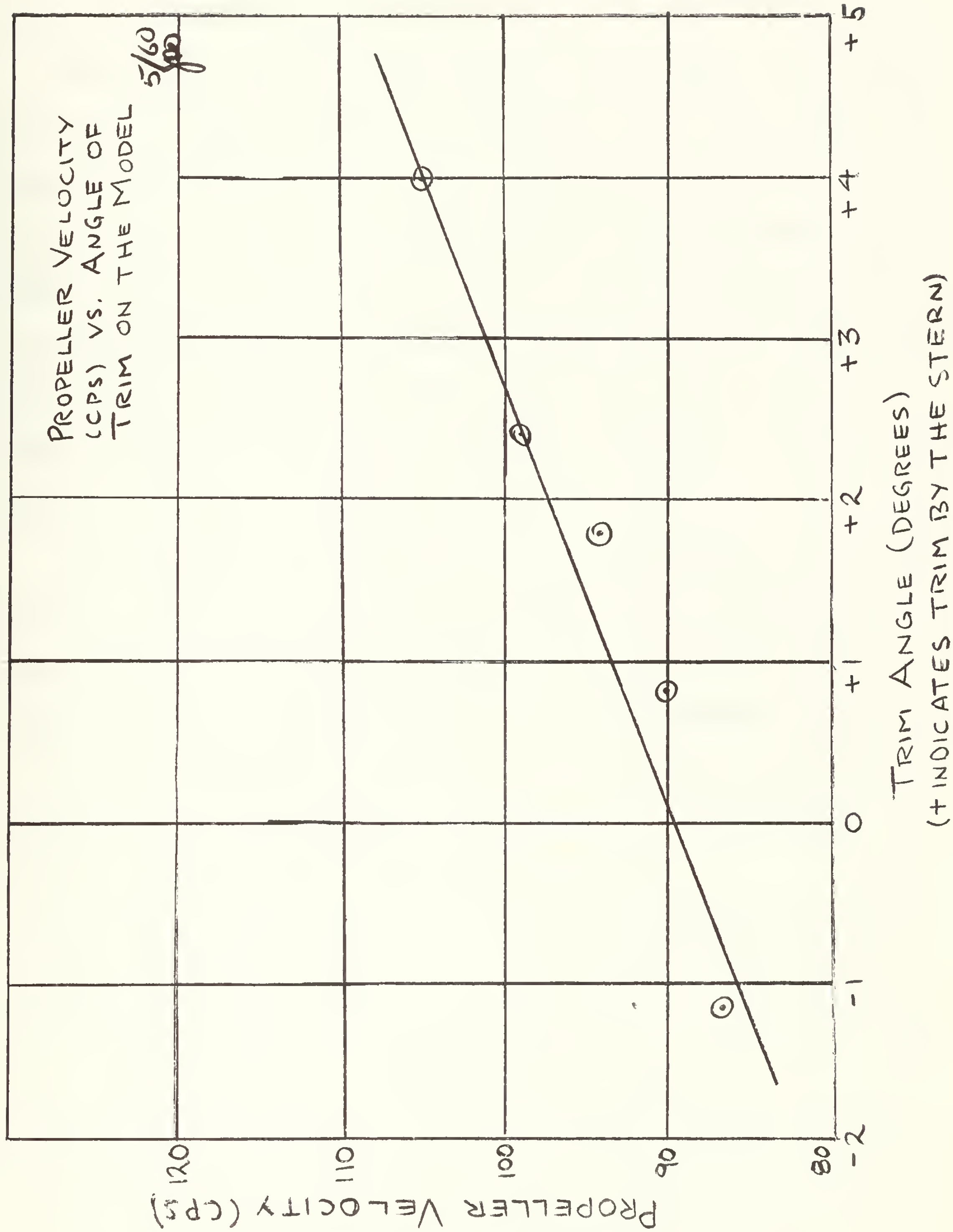
Tests were made by varying the trim and displacement of the model towed in calm water. Displacement changes had little or no noticeable effect on the propeller RPM. On the other hand, trimming the model about its calm water waterline did greatly alter the steady state RPM recorded. This relationship is shown in Figure E-1.

To insure that this variation was applicable to the wake measurement only, and was not an orientation characteristic of the propeller, similar runs with the model trimmed were made with the propeller located at the bow in the calibrating position. In this location, varying the angle of the shaft with respect to the waterline had practically no effect on the RPM. What effect there was was a small decrease in RPM, which should be expected if the normal component of velocity is reduced.

This variation of wake with trim can only be explained in calm water by considering the form of the streamlines in the vicinity of the hull, and particularly in the region of the propeller. When the model is trimmed by the stern, the streamlines,which in an undisturbed region are parallel to the water surface, are less affected by the presence of the hull than when the model is trimmed by the bow. Trimming

FIGURE E-I

Wake Changes with Trim



by the bow "masks" the propeller area, particularly in the case of a very full form such as the model used in these tests.

Extending this explanation to a rough water situation would be extremely involved. For any kind of accuracy it would be necessary to perform an extensive investigation into the flow pattern around a model hull in rough water. Even in calm water this would be a complicated procedure. In waves the streamlines are no longer parallel to the surface, even when not affected by the hull form. They are time varying, both in magnitude and direction, because of the orbital velocities of the waves.

Investigation of this phenomena would require a point by point measurement of both relative fluid velocity and direction. A hot wire or thermistor measurement device might prove to be adaptable for such measurements, though at present there is no evidence that the directional properties of such instrumentation would be sufficiently accurate to give the information required.

To use the results shown in Figure E-I requires that the principle of superposition be accepted. Without more detailed information on the flow pattern around a pitching ship, use of the principle is the only way of utilizing this information. Appendix F will be an attempt to explain the phase and magnitude variations of the results, and the variation of wake with model trim will be included.

This procedure seems justified because the magnitude of the wake changes for small angles of trim is of the same order of magnitude as that expected from orbital velocities of waves.

It is felt that the question of flow pattern around a ship's hull provides an area for much further experimental work, and that the question of propeller wake variation will never be completely explained for the rough water condition until such a study has been made.

APPENDIX F

Correction of Results

It was hoped that, after the frequency response characteristics and the effect of stern motion had been investigated, it would be possible to correct the results of the model runs and show essential agreement with original concept. This now does not seem possible. As examples, two sets of runs will be discussed, and possible corrections outlined.

F-(1) Runs 47-48 (Wave length 3.15 feet)

These short wave length runs are the easiest to correct. This is primarily because there is little or no model motion to account for. If the theory is correct, the only wake variation may be attributed to orbital velocity of the wave.

It is necessary to take into account the phase lag in the propeller system. The analysis in Appendix D indicates that a first order effect is likely, which means a maximum phase shift of 90°. This also checks with the experimentally determined points of Figure D-II, which show a phase shift of about 80° in the vicinity of a frequency of encounter of 2.28 cycles per second. In the actual run, the maximum wake

lagged behind the wave crest by 106°. Applying a 80°-90° correction leaves only a 16°-26° phase lag to be accounted for.

In magnitude, the maximum orbital velocity of the 3.15 feet, 1.9 inches waves was 0.63 feet per second. The wake variation amplitude recorded was 0.089 feet per second. If a phase lag of 80°-90° is assumed to exist in the measurement system, then amplitude attenuation must also be included. In this case it would be about 4 DG, or the ratio of Vw recorded to Vw actual would be 0.4. Correcting the recorded Vw by this factor gives a maximum wake variation amplitude of 0.225 feet per second. If the orbital velocity is corrected for depth of the propeller centerline below the surface (2.25 inches), the magnitude of wake variation measured is in closer agreement with the center-line value of orbital velocity (0.433 ft/sec).

<u>F-(2) Runs 31-32</u> (Wave Length 7.10 feet)

In the case of relatively long wave lengths (in this case 7.10 feet) correcting the results requires consideration of model motion. The results in Table II show that there was a computed 180° phase difference between peak wave velocity and wave crest position. The frequency of encounter was 1.10 cps, and from Figure D-II only about 50° of this phase difference may

be attributed to the characteristics of the propeller system. This leaves about 130° unaccounted for.

Appendix E discussed the variation in wake velocity with trim. In these runs, a large trim angle occurred approximately at the same time the trough was at the stern. The maximum possible trim angle under these conditions was about 7° by the stern, and from Figure E-I, this would be equivalent to a wake variation amplitude of about 0.28 feet per second. This is of the same order of magnitude as the orbital velocity.

The exact time of the maximum pitch angle is not known, because the motion recorded is both pitch and heave. It is possible to estimate whether it is reasonable to assume that both wave and ship motion effects are present.

If the response (wake variation) due to orbital velocity alone is assumed to be of the form:

$$V_{wo} = A \sin \omega t \qquad \text{(F-1)}$$

and the response due to the model pitch angle is assumed to be of the form:

$$V_{wp} = B \sin (\omega t - \phi) \qquad \text{(F-2)}$$

(where ϕ is the angle between the peaks of the two separate responses), it is possible using the principle of superposition to combine these two responses to obtain the total wake variation:

$$V_w = V_{wo} + V_{wp} = A \sin \omega t + \beta \sin (\omega t - \phi) \qquad \text{(F-3)}$$

The results (Table II) show that the recorded wake variation is approximately one and one-half times the maximum orbital velocity. Using this relationship, V_w must be of the form:

$$V_w = 1.5A \sin (\omega t - \theta) \qquad \text{(F-4)}$$

Equations (F-3) and (F-4) may be combined and rewritten as follows:

$$V_w = 1.5A \epsilon^{j(\omega t - \theta)} = A \epsilon^{j\omega t} + B \epsilon^{j(\omega t - \phi)} \qquad \text{(F-5)}$$

It has already been determined that the angle θ, after phase lag correction, is approximately 130°. It remains to determine if the value of ϕ required to satisfy this equation agrees with the known ship motion results.

Rewriting equation (F-5):

$$-0.96A - j\,1.15A = A + (A_1 + j\,A_2) \qquad \text{(F-6)}$$

where $B^2 = A_1^2 + A_2^2$

Solving equation (F-6) gives:

$$A_1 = -1.96\ A$$

$$A_2 = -1.15\ A$$

Or that the motion excitation would have to be:

$$2.3A\,\varepsilon^{j(\omega t-150^\circ)} \quad \text{or} \quad 2.3A \sin(\omega t-150^\circ) \qquad \text{(F-7)}$$

This development assumes that the resulting wake variation is about 1.5 times the orbital velocity amplitude, which was taken from Table II. The resulting phase angle of 150° is in agreement with the motion (stern down to wave crest at prop) phase of -168°.

This can only show that there is a possibility that these two causes may be superimposed to give the actual response. The same method could be used with all the runs, but it is believed that the accuracy of the corrections, and particularly the phase relationships, is so vague that there is little reason for continuing beyond the given examples.

F-(3) Conclusions

Applying these corrections to the results can only serve to indicate that the results recorded were reasonable in the light of the errors found in the measurement systems. For an effective system to determine wake variation, it will be necessary to insure two things:

(1) The phase lag in the wake measurement device must approach either zero, or its characteristics must be accurately known.

(2) Effect of the motion of the stern and/or the attitude of the model hull on wake variation under all conditions must be accurately known before this effect may be separated from the effect of the orbital velocity of the water particles.

APPENDIX G

Specifications

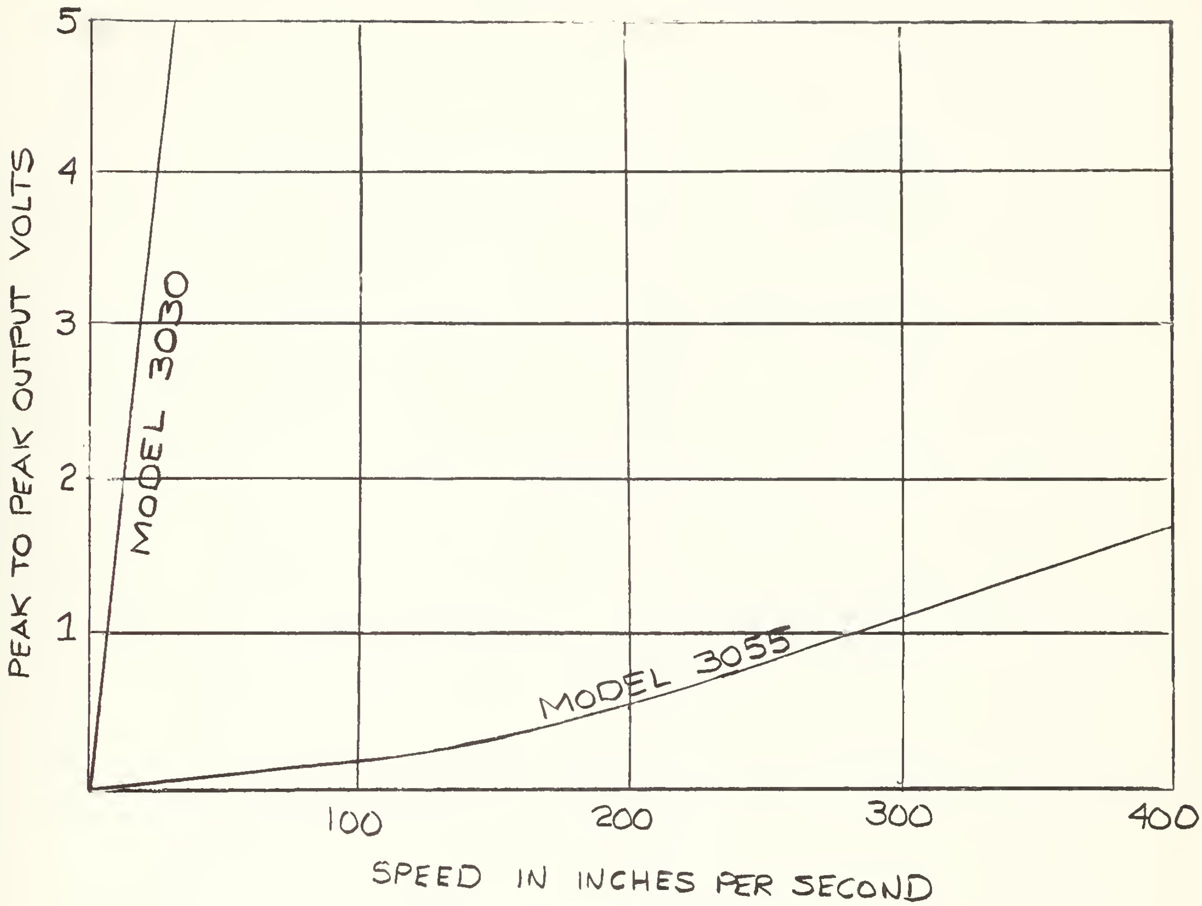


FIGURE G-I

Pickup Output (18)

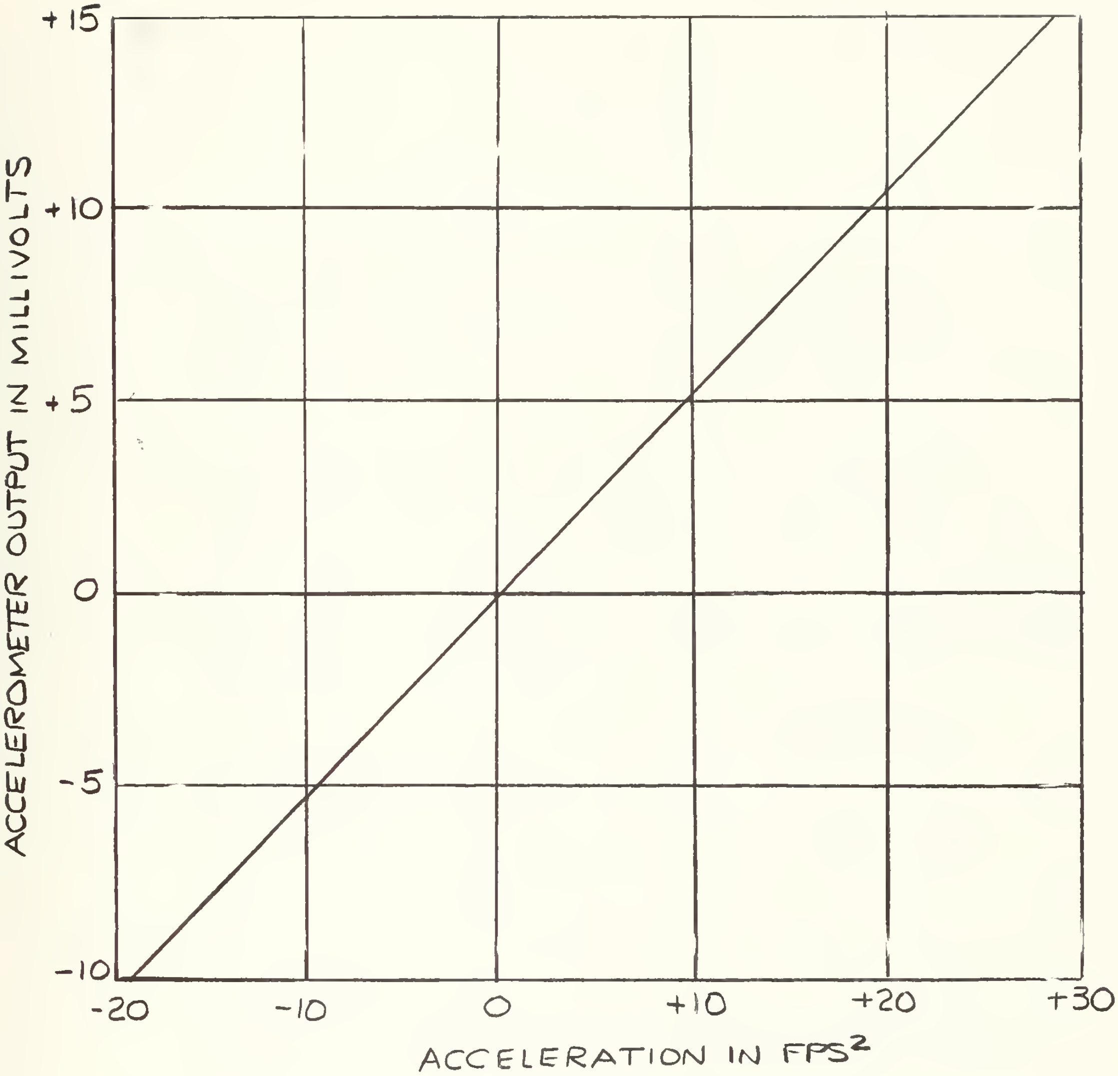
ELECTRO High Sensitivity Magnetic Pickup, Model 3030-AN

Size: 2¼" long x 3/4" diameter
Mounting thread: 5/8" - 18
Impedance: 3400 ohms ± 20% at 1000 cps
Resonant frequency: Over 85 KC
Temperature range: -100° to +225° F.
Weight: 1.89 oz.

ELECTRO Subminiature Magnetic Pickup, Model 3055

Size: 1" long x 9/32" diameter
Mounting thread: 1/4"- 40
Impedance: 100 ohms ± 10% at 1000 cps
Resonant frequency: Over 300 KC
Temperature range: -300° to +500° F.
Weight: 0.12 oz.

FIGURE G-2 (6)



STATHAM Linear Accelerometer, Model A5a-15-350 (19)

Serial number: 6429
Calibration Factor: 2.787 millivolts/volt per G
Range: ± 15 G
Approximate natural frequency: 300 cps
Excitation: 11 volts DC or AC
Weight: 4 oz.

APPENDIX H

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